

Effect of mechanical force field on the electromechanical stability of dielectric elastomers

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To study the electromechanical stability of dielectric elastomer subjected to a mechanical force field, we use free energy functions of variable forms to analyze the mechanical performance of dielectric elastomer. The relation among critical nominal electric field, critical true electric field, nominal stress and mechanical force field is derived. These calculations agree well with the experimental results. The results can help us better understand the stability conditions of dielectric elastomers and furthermore guide the design and manufacture of sensors and actuators based on dielectric elastomers.

dielectric elastomer, mechanical force field, stability analysis

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Nomenclature

L_1, L_2, L_3	original sides of dielectric elastomer
F_1, F_2	mechanical forces
U	electrical voltage
Q	electric charge
$\lambda_1, \lambda_2, \lambda_3$	stretch ratio
l_1, l_2, l_3	dielectric elastomer sides after deformation
s_1, s_2	nominal stress
E^-	nominal electric field
D^-	nominal displacement
σ_1, σ_2	true stress
E	true electric field
D	true displacement
E_{\max}^-	critical nominal electric field
h_c	critical thickness strain
λ_c	critical stretch ratio
E_{\max}	critical true electric field
ε	relative permittivity

$C_1, C_2, \mu_p, \alpha_p$	material constants
k	material constant ratio

1 Introduction

The electromechanical stability theory of dielectric elastomers was first proposed by Suo and Zhao [1–5]. Recent years have seen extensive and in-depth studies of the stability analysis of dielectric elastomer [6–28]. When a voltage is applied on the dielectric elastomer film, the voltage will cause the film to become thinner [6–16, 29–38]. As a result, the same voltage will induce a higher electric field and further cause the dielectric elastomer film to become thinner. When the electric field reaches the critical breakdown electric field, the resulting breakdown will occur in the dielectric elastomer. This process is called electromechanical instability or pull-in instability [3,4].

Zhao and Suo [1,2] proposed that free energy function of any form can be used to analyze the electromechanical stability of dielectric elastomer. As a special case, the elastic strain energy function with one material constant was used

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to analyze the stability of an ideal elastic elastomer under equal biaxial pre-stresses and unequal biaxial pre-stresses. The results illustrated the relation between the nominal electric displacement and the nominal electric field. They showed theoretically that pre-stretch can enhance the stability of dielectric elastomer, which agrees with the experimental tests.

Norris applied the Ogden elastic strain energy model to analyze the stability of the dielectric elastomer [7]. The relation among the critical electric field, the nominal strain and the pre-stretch of dielectric elastomer was obtained accurately. Simultaneously, as a special case, the Neo-Hookean elastic strain energy model, which was a simplified version of the Ogden model, was introduced to give more concise but accurate results.

Further research on the stability of Neo-Hookean type dielectric elastomer was done by Díaz-Calleja's group [8]. The Hessian matrix of dielectric elastomer under two special conditions was deduced. Furthermore, the stable domain and unstable domain of dielectric elastomer were described. These results can help us understand the stability performance of Neo-Hookean type dielectric elastomer more thoroughly.

Ref. [10] applied the elastic strain energy function with two material constants to analyze the stability performance of dielectric elastomers. The introduced material constant ratio k could help us analyze the stability of various dielectric elastomers. The relation between the nominal electric displacement and nominal electric field of various dielectric elastomers was derived.

In this paper, variable free energy functions are applied to analyze the mechanical performance of dielectric elastomer subjected to a mechanical force field. The relation among critical nominal electric field, critical true electric field, nominal stress and mechanical force field is derived, which agrees well with the experimental results.

2 Stability parameters of electromechanical coupling system

As is shown in Figure 1, in the referenced state, the mem-

brane of dielectric elastomer is of sides L_1 , L_2 , and L_3 . When the dielectric elastomer is subject to mechanical forces F_1 , F_2 , and to electrical voltage U , the three sides of the membrane deform to l_1 , l_2 and l_3 , and electric charge Q flows through the external circuit from one electrode to another. Here λ_i , $i=1, 2, 3$ represents the principal stretch ratio after deformation, and s_1, s_2 are the nominal stress, which can be derived in the undeformed state by dividing the pre-stretch force by the area, i.e., $s_1=F_1/L_2L_3$, $s_2=F_2/L_1L_3$. The dielectric elastomer is taken to be incompressible, so that $\lambda_1=1/\lambda_1\lambda_2$. Similarly, the nominal electric field $E^*=U/L_e$ is defined as the voltage divided by the original thickness, and the nominal electric displacement $D^*=Q/L_1L_2$ is defined as the charge divided by the area before deformation. The corresponding true electric field can be obtained by dividing the electric voltage U by the thickness in the deformed state, i.e., $E=U/\lambda_3L_3$, and also the corresponding true electric displacement is $D=Q/\lambda_1\lambda_2L_2$.

In this paper, the effect of mechanical force field on the electromechanical stability of dielectric elastomers is studied. Stability performance parameters are defined and the mathematical expressions of those parameters are derived. Here we define the critical nominal electric field E_{\max}^* , the critical true electric field E_{\max} , the nominal stress s , the true stress σ , the critical stretch ratio λ_c , the critical thickness strain $h_e=1-1/\lambda_c^2$ as the stability performance parameters, where the critical quantities and the true quantities represent respectively the different physical performance from the stable states to the unstable states, while the ratio variables denote the deformation comportment.

3 The mathematical expression of system stability parameters

To the electromechanical coupling system of the dielectric elastomer, elastic strain energy function (mechanical force field) and the electric energy destiny function (electric field) can be coupled. Then the free energy function of the coupling system is obtained [15]:

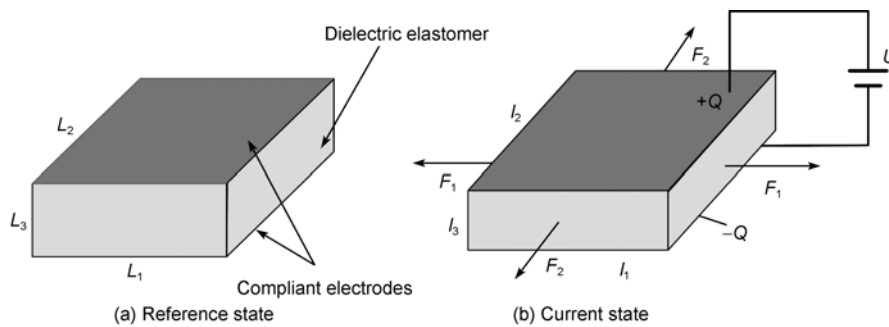


Figure 1 The dielectric elastomer is sandwiched between two compliant electrodes. (a) In the reference state, the dielectric is subjected to neither forces nor voltage; (b) in the current state, subject to forces and voltage, the dielectric elastomer deforms.

$$W(\lambda_1, \lambda_2, \lambda_1^{-1}\lambda_2^{-1}, D^-) = U(\lambda_1, \lambda_2, \lambda_1^{-1}\lambda_2^{-1}) + V(\lambda_1, \lambda_2, \lambda_1^{-1}\lambda_2^{-1}, D^-), \quad (1)$$

where $W(\lambda_1, \lambda_2, \lambda_1^{-1}\lambda_2^{-1}, D^-)$ denotes the free energy function of dielectric elastomer electromechanical coupling system, $U(\lambda_1, \lambda_2, \lambda_1^{-1}\lambda_2^{-1})$ represents the elastic strain energy function, and $V(\lambda_1, \lambda_2, \lambda_1^{-1}\lambda_2^{-1}, D^-)$ reveals the electric energy destiny function. The Hessian matrix is

$$H = \begin{bmatrix} U_{11} + V_{11} & U_{12} + V_{12} & V_{13} \\ U_{12} + V_{12} & U_{22} + V_{22} & V_{23} \\ V_{13} & V_{23} & V_{33} \end{bmatrix}, \quad (2)$$

where

$$U_{ij} = \frac{\partial U}{\partial \lambda_i \partial \lambda_j}, \quad V_{ij} = \frac{\partial V}{\partial \lambda_i \partial \lambda_j}, \quad i, j = 1, 2,$$

$$V_{13} = \frac{\partial V}{\partial \lambda_1 \partial D^-}, \quad V_{23} = \frac{\partial V}{\partial \lambda_2 \partial D^-}, \quad V_{33} = \frac{\partial V}{\partial D^{-2}}.$$

The elastic strain energy function above represents the effect of mechanical force field on the dielectric elastomer electromechanical coupling system, and the electric energy destiny function represents the effect of electric field on the dielectric elastomer electromechanical coupling system. For the effect of mechanical force field, the electric energy destiny function can be set as a constant value. So the model of the electric energy destiny function with the dielectric constant ε invariable can be expressed as follows:

$$V(\lambda_1, \lambda_2, \lambda_1^{-1}\lambda_2^{-1}, D^-) = \frac{D^{-2}}{2\varepsilon} \lambda_1^{-2} \lambda_2^{-2}. \quad (3)$$

Inserting eq. (3) into eq. (2), we obtain the Hessian matrix of dielectric elastomer:

$$H = \begin{bmatrix} U_{11} + \frac{3D^{-2}}{\varepsilon} \lambda_1^{-4} \lambda_2^{-2} & U_{12} + \frac{2D^{-2}}{\varepsilon} \lambda_1^{-3} \lambda_2^{-3} & -\frac{2D^-}{\varepsilon} \lambda_1^{-3} \lambda_2^{-2} \\ U_{12} + \frac{2D^{-2}}{\varepsilon} \lambda_1^{-3} \lambda_2^{-3} & U_{22} + \frac{3D^{-2}}{\varepsilon} \lambda_2^{-4} \lambda_1^{-2} & -\frac{2D^-}{\varepsilon} \lambda_2^{-3} \lambda_1^{-2} \\ -\frac{2D^-}{\varepsilon} \lambda_1^{-3} \lambda_2^{-2} & -\frac{2D^-}{\varepsilon} \lambda_2^{-3} \lambda_1^{-2} & \frac{1}{\varepsilon} \lambda_1^{-2} \lambda_2^{-2} \end{bmatrix}. \quad (4)$$

And then the determinant of Hessian matrix is

$$\det(H) = \frac{U_{11}U_{22}}{\varepsilon} \lambda_1^{-2} \lambda_2^{-2} - \frac{U_{11}D^{-2}}{\varepsilon^2} \lambda_1^{-4} \lambda_2^{-6} + \frac{3D^{-2}U_{22}}{\varepsilon^2} \lambda_1^{-6} \lambda_2^{-4} - \frac{3D^{-4}}{\varepsilon^3} \lambda_1^{-8} \lambda_2^{-8} - \frac{U_{12}^2}{\varepsilon} \lambda_1^{-2} \lambda_2^{-2}$$

$$+ \frac{4D^{-4}}{\varepsilon^3} \lambda_1^{-8} \lambda_2^{-8} + \frac{4U_{12}}{\varepsilon^2} \lambda_1^{-5} \lambda_2^{-5} + \frac{8D^{-4}}{\varepsilon^3} \lambda_1^{-8} \lambda_2^{-8} - \frac{4D^{-2}U_{22}}{\varepsilon^2} \lambda_1^{-6} \lambda_2^{-4} - \frac{12D^{-4}}{\varepsilon^3} \lambda_1^{-8} \lambda_2^{-8}. \quad (5)$$

For such a dielectric elastomer electromechanical coupling system, we have $D^- = E^- \varepsilon \lambda_1^2 \lambda_2^2$ and $E = E^- \lambda_1 \lambda_2$, where E^- represents the nominal electric field, and E denotes the true electric field. Inserting them into eq. (5), the accurate relation of critical nominal breakdown electric field E_{\max}^- , critical true breakdown electric field E_{\max} , nominal strain and mechanical force field on the dielectric elastomer are obtained as follows:

$$\varepsilon E_{\max}^{-2} = \frac{1}{6} \left[4U_{12} \lambda_1^{-1} \lambda_2^{-1} - \lambda_2^{-2} U_{11} - \lambda_1^{-2} U_{22} + \sqrt{(\lambda_2^{-2} U_{11} + \lambda_1^{-2} U_{22} - 4U_{12} \lambda_1^{-1} \lambda_2^{-1})^2 + 12 \lambda_1^{-2} \lambda_2^{-2} (U_{11} U_{22} - U_{12}^2)} \right], \quad (6)$$

$$\varepsilon E_{\max}^2 = \frac{1}{6} \left[4U_{12} \lambda_1 \lambda_2 - \lambda_1^2 U_{11} - \lambda_2^2 U_{22} + \sqrt{(\lambda_1^2 U_{11} + \lambda_2^2 U_{22} - 4U_{12} \lambda_1 \lambda_2)^2 + 12 \lambda_1^2 \lambda_2^2 (U_{11} U_{22} - U_{12}^2)} \right], \quad (7)$$

$$s_j = U_j - \lambda_j^{-1} \varepsilon E^2, \quad (8)$$

$$\sigma_j = \lambda_j U_j - \varepsilon E_{\max}^2, \quad (9)$$

$j=1, 2$. Let $s_j = 0$, and the maximum stretch λ_c can be calculated. Inserting this result into eqs. (6) and (7), we can get the critical nominal electric field E_{\max}^- , and the critical true electric field E_{\max} . Meanwhile we can get the critical thickness strain $h_\varepsilon = 1 - 1/\lambda_c^2$, which represents the maximum deformation of dielectric elastomer under the coupling of electric field and mechanical field.

To simplify the above relations, we take into account the special condition of equal biaxial pre-stretch of dielectric elastomer, namely, $\lambda_1 = \lambda_2 = \lambda$, then $U_{11} = U_{22}$, and inserting them into eqs. (6)–(9), we obtain:

$$\varepsilon E_{\max}^{-2} = \frac{\lambda^{-2}}{3} (U_{11} + U_{12}), \quad (10)$$

$$\varepsilon E_{\max}^2 = \frac{\lambda^2}{3} (U_{11} + U_{12}), \quad (11)$$

$$U_1 - \frac{\lambda}{3} (U_{11} + U_{12}) = s, \quad (12)$$

$$U_1 \lambda - \frac{\lambda^2}{3} (U_{11} + U_{12}) = \sigma, \quad (13)$$

$$h_e = 1 - 1/\lambda_c^2. \quad (14)$$

Eqs. (10)–(14) denote the accurate relations of critical nominal electric field, critical true electric field, nominal strain and elastic strain energy function (effect of mechanical force field) under the condition of equal biaxial pre-stretch.

4 The effect of mechanical force field

The dielectric elastomer electromechanical stability is analyzed in detail in this part when the dielectric elastomer is subjected to a mechanical force field. When different elastic strain energy functions are introduced, the explicit expression of the dielectric elastomer performance parameters is obtained under the condition of equal biaxial pre-stretch. By introducing the model of elastic strain energy function with

$$H = \begin{bmatrix} \mu(1 + 3\lambda_1^{-4}\lambda_2^{-2}) + \frac{3D^{-2}}{\varepsilon}\lambda_1^{-4}\lambda_2^{-2} & 2\mu\lambda_1^{-3}\lambda_2^{-3} + \frac{2D^{-2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2D^{-2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} \\ 2\mu\lambda_1^{-3}\lambda_2^{-3} + \frac{2D^{-2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & \mu(1 + 3\lambda_2^{-4}\lambda_1^{-2}) + \frac{3D^{-2}}{\varepsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2D^{-2}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} \\ -\frac{2D^{-2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2D^{-2}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix}. \quad (19)$$

The special condition of dielectric elastomer loading is taken into account, namely $\lambda_1 = \lambda_2 = \lambda$. We have $U_{11} = U_{22}$, and inserting it into eqs. (10)–(13), we get:

$$\varepsilon E_{\max}^{-2} = \frac{\mu}{3}(\lambda^{-2} + 5\lambda^{-8}), \quad (20)$$

$$\varepsilon E_{\max}^2 = \frac{\mu}{3}(\lambda^2 + 5\lambda^{-4}), \quad (21)$$

$$s = \frac{2\mu}{3}(\lambda - 4\lambda^{-5}), \quad (22)$$

$$\sigma = \frac{2\mu}{3}(\lambda^2 - 4\lambda^{-4}). \quad (23)$$

For eq. (22), let $s=0$. The stretch λ reaches its critical value λ_{\max} , then $\lambda_{\max} = 1.26$. By inserting $\lambda_{\max} = 1.26$ into eqs. (20) and (21), critical nominal electric field and critical true electric field are obtained, namely $E_{\max}^- = 0.69\sqrt{\mu/\varepsilon}$, $E_{\max} = 1.09\sqrt{\mu/\varepsilon}$. Inserting them into eq. (14), then the value of the critical thickness strain h_e is about 37%. Clearly, the results coincide well with Suo's conclusion [1].

Figure 2 gives the relations between electromechanical stability parameters and the stretch of dielectric elastomer, including critical nominal electric field, critical true electric

one material constant, the free energy function can be written as:

$$W(\lambda_1, \lambda_2, D^-) = \frac{\mu}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2} - 3) + \frac{D^{-2}}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \quad (15)$$

$$s_1 = \frac{\partial W}{\partial \lambda_1} = \mu(\lambda_1 - \lambda_1^{-3}\lambda_2^{-2}) - \frac{D^{-2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \quad (16)$$

$$s_2 = \frac{\partial W}{\partial \lambda_2} = \mu(\lambda_2 - \lambda_2^{-3}\lambda_1^{-2}) - \frac{D^{-2}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2}, \quad (17)$$

$$\tilde{E} = \frac{\partial W}{\partial D^-} = \frac{D^-}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \quad (18)$$

Inserting the above equations into eq. (4), the Hessian matrix of dielectric elastomer is

field, critical nominal stress, and critical true stress.

By introducing the model of elastic strain energy function with two material constants, the free energy function can be expressed as follows:

$$W(\lambda_1, \lambda_2, D^-) = \frac{C_1}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2} - 3) + \frac{C_2}{2}(\lambda_1^{-2} + \lambda_2^{-2} + \lambda_1^2\lambda_2^2 - 3) + \frac{D^{-2}}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \quad (24)$$

$$s_1 = \frac{\partial W}{\partial \lambda_1} = C_1(\lambda_1 - \lambda_1^{-3}\lambda_2^{-2}) + C_2(-\lambda_1^{-3} + \lambda_1\lambda_2^2) - \frac{D^{-2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \quad (25)$$

$$s_2 = \frac{\partial W}{\partial \lambda_2} = C_1(\lambda_2 - \lambda_2^{-3}\lambda_1^{-2}) + C_2(-\lambda_2^{-3} + \lambda_2\lambda_1^2) - \frac{D^{-2}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2}, \quad (26)$$

$$E^- = \frac{\partial W}{\partial D^-} = \frac{D^-}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \quad (27)$$

Substituting it into eq. (4), the Hessian matrix of dielectric elastomer can be written as follows:

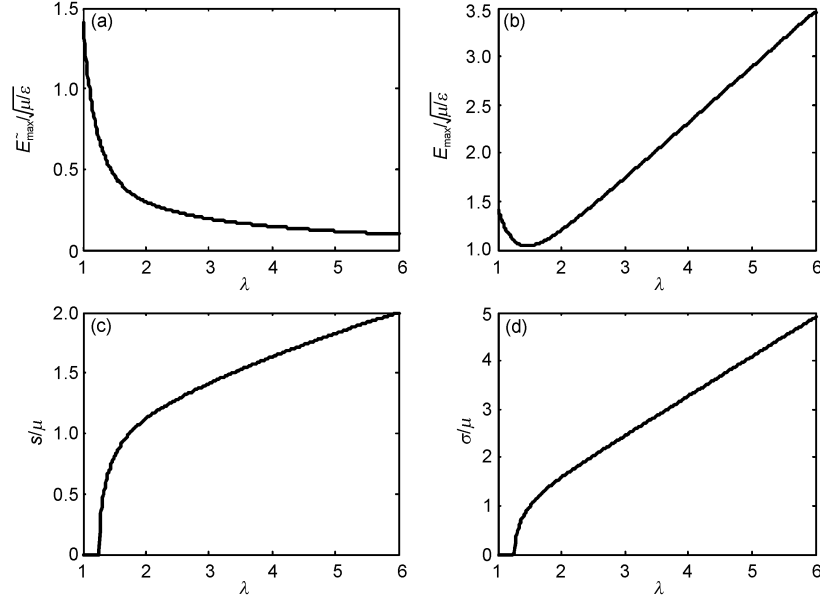


Figure 2 Relations between electromechanical stability parameters and the stretch of the dielectric elastomer. (a) Critical nominal electric field; (b) critical true electric field; (c) critical nominal stress; (d) critical true stress.

$$H = \begin{bmatrix} C_1(1+3\lambda_1^{-4}\lambda_2^{-2}) + C_2(3\lambda_1^{-4} + \lambda_2^2) + \frac{3D^{-2}}{\epsilon}\lambda_1^{-4}\lambda_2^{-2} & 2C_1\lambda_1^{-3}\lambda_2^{-3} + 2C_2\lambda_1\lambda_2 + \frac{2D^{-2}}{\epsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2D^{-2}}{\epsilon}\lambda_1^{-3}\lambda_2^{-2} \\ 2C_1\lambda_1^{-3}\lambda_2^{-3} + 2C_2\lambda_1\lambda_2 + \frac{2D^{-2}}{\epsilon}\lambda_1^{-3}\lambda_2^{-3} & C_1(1+3\lambda_2^{-4}\lambda_1^{-2}) + C_2(3\lambda_2^{-4} + \lambda_1^2) + \frac{3D^{-2}}{\epsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2D^{-2}}{\epsilon}\lambda_2^{-3}\lambda_1^{-2} \\ -\frac{2D^{-2}}{\epsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2D^{-2}}{\epsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\epsilon}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix}. \quad (28)$$

Considering the special condition of dielectric elastomer loading, namely $\lambda_1 = \lambda_2 = \lambda$, then $U_{11} = U_{22}$, and substituting it into eqs. (10)–(14), we have:

$$\epsilon E_{\max}^{-2} = \frac{C_1}{3}(\lambda^{-2} + 5\lambda^{-8}) + C_2(\lambda^{-6} + 1), \quad (29)$$

$$\epsilon E_{\max}^2 = \frac{C_1}{3}(\lambda^2 + 5\lambda^{-4}) + C_2(\lambda^{-2} + \lambda^4), \quad (30)$$

$$s = \frac{2C_1}{3}(\lambda - 4\lambda^{-5}) - 2C_2\lambda^{-3}, \quad (31)$$

$$\sigma = \frac{2C_1}{3}(\lambda^2 - 4\lambda^{-4}) - 2C_2\lambda^{-2}. \quad (32)$$

For convenience, we introduce a dimensionless quantity k which depends on the material and the activated shape of the dielectric elastomer, where λ_1 , when λ_2 is a constant, as 0, L_1 , L_2 , L_3 . For F_1, F_2 , let $s=0$. The critical stretch can be evaluated to be $\lambda_c=1.48$. Inserting it into eq. (29), the maximum value of the nominal electric field $E_{\max}^{\sim} = 1.148\sqrt{C_1/\epsilon}$ is obtained. Substituting it into eq. (30), the

maximum value of the true electric field $E_{\max} = 2.514\sqrt{C_1/\epsilon}$ is obtained.

For $k=1/2$, let $s=0$, and the critical stretch can be evaluated to be $\lambda_c=1.37$. Inserting it into eq. (29), the maximum value of the nominal electric field $E_{\max}^{\sim} = 0.936\sqrt{C_1/\epsilon}$ is obtained. Inserting it into eq. (30), the maximum value of the true electric field $E_{\max} = 1.757\sqrt{C_1/\epsilon}$ is obtained.

For $k=1/4$, let $s=0$, and the critical stretch can be evaluated to be $\lambda_c=1.32$. Inserting it into eq. (29), the maximum value of the nominal electric field $E_{\max}^{\sim} = 0.817\sqrt{C_1/\epsilon}$ is obtained. Inserting it into eq. (30), the maximum value of the true electric field $E_{\max} = 1.423\sqrt{C_1/\epsilon}$ is obtained.

For $k=1/5$, let $s=0$, and the critical stretch can be evaluated to be $\lambda_c=1.32$. Inserting it into eq. (29), the maximum value of the nominal electric field $E_{\max}^{\sim} = 0.792\sqrt{C_1/\epsilon}$ is obtained. Inserting it into eq. (30), the

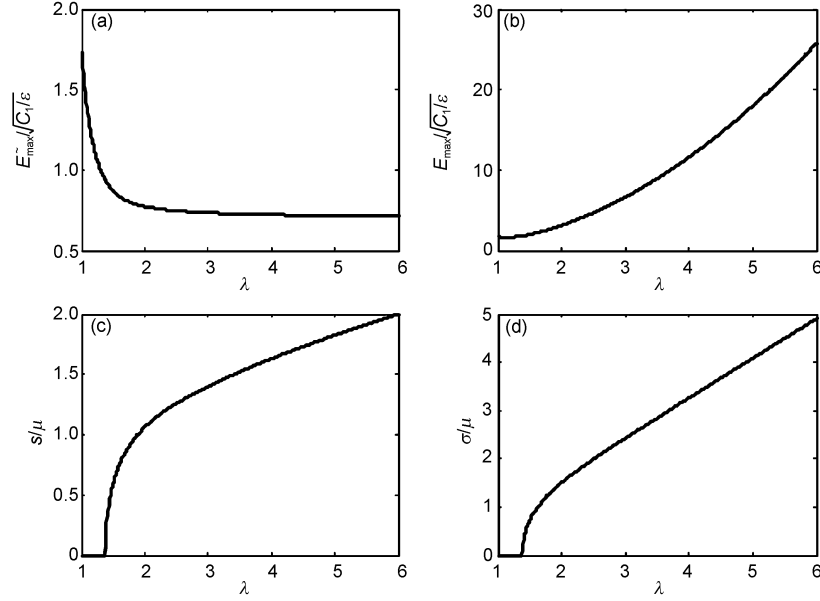


Figure 3 Relations between stability parameters and the stretch when $k=0.5$. (a) Critical nominal electric field; (b) critical true electric field; (c) critical nominal stress; (d) critical true stress.

maximum value of the true electric field $E_{\max} = 1.338\sqrt{C_1/\varepsilon}$ is obtained. The results correspond to Liu's conclusion [5].

Figure 3 shows the relations between electromechanical stability parameters (critical nominal electric field, critical true electric field, critical nominal stress and critical true stress) and the stretch of dielectric elastomer under equal biaxial stresses and material constant ratio $k=1/2$.

For the model of elastic strain energy function with various material constants, the special condition of dielectric elastomer is taken into account, namely $\lambda_1 = \lambda_2 = \lambda$, then $U_{11} = U_{22}$. Inserting it into eqs. (6)–(8), we get

$$W(\lambda_1, \lambda_2, D^-) = \sum_{p=1}^N \frac{\mu_p}{\alpha_p} (\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_1^{-\alpha_p} \lambda_2^{-\alpha_p} - 3)$$

$H =$

$$\begin{bmatrix} \sum_{p=1}^N \mu_p [(\alpha_p - 1) \lambda_1^{\alpha_p - 2} + (\alpha_p + 1) \lambda_1^{-\alpha_p - 2} \lambda_2^{-\alpha_p}] + \frac{3D^{-2}}{\varepsilon} \lambda_1^{-4} \lambda_2^{-2} & \sum_{p=1}^N \mu_p \alpha_p \lambda_1^{-\alpha_p - 1} \lambda_2^{-\alpha_p - 1} + \frac{2D^{-2}}{\varepsilon} \lambda_1^{-3} \lambda_2^{-3} & -\frac{2D^-}{\varepsilon} \lambda_1^{-3} \lambda_2^{-2} \\ \sum_{p=1}^N \mu_p \alpha_p \lambda_1^{-\alpha_p - 1} \lambda_2^{-\alpha_p - 1} + \frac{2D^{-2}}{\varepsilon} \lambda_1^{-3} \lambda_2^{-3} & \sum_{p=1}^N \mu_p [(\alpha_p - 1) \lambda_2^{\alpha_p - 2} + (\alpha_p + 1) \lambda_2^{-\alpha_p - 2} \lambda_1^{-\alpha_p}] + \frac{3D^{-2}}{\varepsilon} \lambda_2^{-4} \lambda_1^{-2} & -\frac{2D^-}{\varepsilon} \lambda_2^{-3} \lambda_1^{-2} \\ -\frac{2D^-}{\varepsilon} \lambda_1^{-3} \lambda_2^{-2} & -\frac{2D^-}{\varepsilon} \lambda_2^{-3} \lambda_1^{-2} & \frac{1}{\varepsilon} \lambda_1^{-2} \lambda_2^{-2} \end{bmatrix}, \quad (37)$$

$$\varepsilon E_{\max}^2 = \frac{1}{3} \sum_{p=1}^N \mu_p [(\alpha_p - 1) \lambda^{\alpha_p - 4} + (2\alpha_p + 1) \lambda^{-2\alpha_p - 4}], \quad (38)$$

$$\varepsilon E_{\max}^2 = \frac{1}{3} \sum_{p=1}^N \mu_p [(\alpha_p - 1) \lambda^{\alpha_p} + (2\alpha_p + 1) \lambda^{-2\alpha_p}], \quad (39)$$

$$+ \frac{D^{-2}}{2\varepsilon} \lambda_1^{-2} \lambda_2^{-2}, \quad (33)$$

$$s_1 = \frac{\partial W}{\partial \lambda_1} = \sum_{p=1}^N \mu_p (\lambda_1^{\alpha_p - 1} - \lambda_1^{-\alpha_p - 1} \lambda_2^{-\alpha_p}) - \frac{D^{-2}}{\varepsilon} \lambda_1^{-3} \lambda_2^{-2}, \quad (34)$$

$$s_2 = \frac{\partial W}{\partial \lambda_2} = \sum_{p=1}^N \mu_p (\lambda_2^{\alpha_p - 1} - \lambda_2^{-\alpha_p - 1} \lambda_1^{-\alpha_p}) - \frac{D^{-2}}{\varepsilon} \lambda_2^{-3} \lambda_1^{-2}, \quad (35)$$

$$E^- = \frac{\partial W}{\partial D^-} = \frac{D^-}{\varepsilon} \lambda_1^{-2} \lambda_2^{-2}. \quad (36)$$

Substituting eq. (33) into eq. (4), the Hessian matrix of dielectric elastomer is

$$\frac{1}{3\lambda} \sum_{p=1}^N \mu_p [(4 - \alpha_p) \lambda^{\alpha_p} - (4 + 2\alpha_p) \lambda^{-2\alpha_p}] = s, \quad (40)$$

$$\frac{1}{3} \sum_{p=1}^N \mu_p [(4 - \alpha_p) \lambda^{\alpha_p} - (4 + 2\alpha_p) \lambda^{-2\alpha_p}] = \sigma. \quad (41)$$

For eq. (40), let $s=0$, and the critical stretch ratio λ_c is obtained. Inserting it into eqs. (38) and (39), then the explicit expressions of critical nominal electric and critical true electric field can be obtained. The Neo-Hookean model and the Mooney-Rivlin model can be regarded as the special case of the Ogden elastic strain energy function with various material constants. The following part is to simplify the Ogden model, and apply the simplified form to evaluate the critical nominal electric field, the critical true electric field and the nominal stress, then compare and validate them with the results obtained by the above models.

Assuming $N=1$, $\alpha_1=\alpha$, $\mu_1=\mu$, the critical stretch can be evaluated to be

$$\lambda_c = \left(\frac{4+2\alpha}{4-\alpha} \right)^{1/3\alpha},$$

$$\varepsilon E_{\max}^{-2} = \frac{1}{3}\mu \left[(\alpha-1) \left(\frac{4+2\alpha}{4-\alpha} \right)^{\frac{\alpha-4}{3\alpha}} + (2\alpha+1) \left(\frac{4+2\alpha}{4-\alpha} \right)^{\frac{-2\alpha-4}{3\alpha}} \right], \quad (42)$$

$$\varepsilon E_{\max}^2 = \frac{1}{3}\mu \left[(\alpha-1) \left(\frac{4+2\alpha}{4-\alpha} \right)^{\frac{1}{3}} + (2\alpha+1) \left(\frac{4+2\alpha}{4-\alpha} \right)^{\frac{2}{3}} \right]. \quad (43)$$

Obviously, assuming $\alpha=2$, the critical stretch $\lambda_c=1.26$. Then inserting it into eqs. (42) and (43), the critical nominal and the critical true electric field can be obtained, namely $E_{\max}^- = 0.69\sqrt{\mu/\varepsilon}$, $E_{\max} = 1.09\sqrt{\mu/\varepsilon}$. For comparison, the results agree well with those by using the Neo-Hookean model.

Assuming $N=2$, $\alpha_2=-\alpha_1=-2$, $\mu_2=m\mu_1$, inserting them into eq. (40), $\lambda^6 + 3m\lambda^2 - 8 = 0$, when m equals different values, the critical stretch ratio λ can be obtained. $m=-1$, $\lambda_c=1.48$. Inserting them into eq. (38), the maximum value of the nominal electric field $E_{\max}^- = 1.148\sqrt{\mu_1/\varepsilon}$. Inserting them into eq. (39), the maximum value of the nominal electric field $E_{\max} = 2.514\sqrt{\mu_1/\varepsilon}$. When m is chosen as $-1/2$, $-1/4$, $-1/5$, respectively, the corresponding critical stretch is evaluated as $\lambda_c=1.37$, 1.32 , 1.30 , the critical nominal electric field $E_{\max}^- = 0.936\sqrt{\mu_1/\varepsilon}$, $0.817\sqrt{\mu_1/\varepsilon}$, $0.792\sqrt{\mu_1/\varepsilon}$ respectively, while the critical true electric field $E_{\max} = 1.757\sqrt{\mu_1/\varepsilon}$,

$1.423\sqrt{\mu_1/\varepsilon}$, $1.338\sqrt{\mu_1/\varepsilon}$ respectively. Let $C_2=-\mu_2$, $C_1=\mu_1$, $k=-m$. The evaluated result coincides with the result when the Mooney-Rivlin model is used.

5 Conclusions

This paper has obtained the accurate relation among critical nominal, critical true electric field, nominal stress and different strain energy functions in a dielectric elastomer electromechanical coupling system. The strain energy functions, which are explicit expressions of systemic stability parameters, of the Neo-Hookean model, the Mooney-Rivlin model and the Ogden model, are derived. Simultaneously, the pre-digested expression of Ogden model is evaluated. And the deduced results agree with the experimental results. There is great instructional significance in actuator design and fabrication.

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