Electromechanical stability of dielectric elastomer

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Electromechanical instability may occur in dielectric elastomer films due to the coupling between mechanical forces and electric fields. According to Zhao and Suo [Appl. Phys. Lett. **91**, 061921 (2007)], free-energy in any form, which consists of elastic strain energy and electric energy, can be used to analyze the electromechanical stability of dielectric elastomer. By taking the permittivity as a variable depending on the deformation in a free energy function, a relationship is established among critical nominal electric field, critical real electric field, nominal stress, and principal stretch ratios. The accurate expressions of these parameters are presented for a special equal biaxial stretch case. All the results obtained by utilizing the single material constant neo-Hookean elastic strain energy model coincide with the conclusions by Zhao and Suo. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3138153]

A layer of a dielectric elastomer reduces in thickness and expands its area when a voltage is applied across its thickness.[1] Such changes cause a higher electric field in the dielectric elastomer, which results in a positive feedback. When the critical electric field is reached, the dielectric elastomer film will break down. Such a failure is known as instability of material, which has a significant influence on the capacity of a dielectric elastomer either as an actuator or sensor.

The research on the failure and nonlinear electromechanical stability of a dielectric elastomer has been one of the most popular subjects in recent years.[2–6] Zhao and Suo[2] proposed a general method using the free energy function of dielectric elastmers to analyze their stability. Their theoretical study proved that the critical electric fields for electromechanical stability of dielectric elastomers can be increased by prestretch. The critical electric fields evaluated by the method[2] are consistent with experimental results.[7] Norris[3] used the elastic strain energy model developed by Ogden to analyze the stability of elastomers. Díaz-Calleja *et al.*[4] made an in-depth investigation of the stability of neo-Hookean silicone and located the instability region of this material. Liu *et al.*[5] studied the stability of dielectric elastomers using an elastic strain energy function with two material constants and found the ratio of the two constants can be used to represent the stability of different types of dielectric elastomer. Recently, we studied the stable domain of Mooney–Rivlin silicone, which provides useful theory guidance for the research on this kind of material.[6]

In this paper, a nonlinear expression of permittivity as a function of the stretch ratio has been proposed. Based on this expression, we analyzed the electromechanical stability of dielectric elastomer. The relationship among critical nominal electric field, critical real electric field, nominal stress, and principal stretch ratios has been established.

Suppose the dielectric elastomer is incompressible, the free energy function for the electromechanical coupling system of a dielectric elastomer can then be expressed as

$$W(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},\tilde{D}) = U(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1}) + V(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},\tilde{D}), \quad (1)$$

where $U(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1})$ and $V(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},\tilde{D})$ are the elastic strain energy and electric field energy density functions, respectively, λ_1 and λ_2 are the two principal stretch ratios, and $\tilde{D}$ is the nominal electric displacement.

The Hessian matrix of the system can be written as

$$H = \begin{bmatrix} U_{11}+V_{11} & U_{12}+V_{12} & V_{13} \\ U_{12}+V_{12} & U_{22}+V_{22} & V_{23} \\ V_{13} & V_{23} & V_{33} \end{bmatrix}, \quad (2)$$

where $U_{ij}=\partial^2 U/\partial\lambda_i\partial\lambda_j$, $V_{ij}=\partial^2 V/\partial\lambda_i\partial\lambda_j$, $i,j=1,2$, $V_{13}=\partial^2 V/\partial\lambda_1\partial\tilde{D}$, $V_{23}=\partial^2 V/\partial\lambda_2\partial\tilde{D}$, $V_{33}=\partial^2 V/\partial\tilde{D}^2$.

According to research of Kofod *et al.*[8] on acrylic, the permittivity of a dielectric elastomer is variable of deformation. We can express the permittivity of acrylic as a function of stretches,

$$\varepsilon(\lambda_1,\lambda_2) = \begin{cases} (a\lambda_1\lambda_2+b)\varepsilon_0, & \lambda_1\lambda_2 \le s, \\ c\varepsilon_0, & \lambda_1\lambda_2 > s, \end{cases} \quad (3)$$

where a, b, c, and s are the speific constants of an elastomer material, $a=-0.016$, $b=4.716$ $c=4.48$by fitting to experimental data,[8] and $\varepsilon_0=8.85\times10^{-12}$ F/m is the permittivity of free space. Equation (3) can be generalized and applied to other dielectric elastomers.

According to Ref. 2, the electric field energy density function can be expressed as $V(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},\tilde{D}) = \tilde{D}^2\lambda_1^{-2}\lambda_2^{-2}/2\varepsilon(\lambda_1,\lambda_2)$.

The free energy function of a dielectric elastomer can be given by

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$$W(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},D^{\sim}) = \begin{cases} U(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1}) + \dfrac{D^{\sim 2}}{2\varepsilon_0(a\lambda_1\lambda_2+b)}\lambda_1^{-2}\lambda_2^{-2}, & \lambda_1\lambda_2 \le s, \\ U(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1}) + \dfrac{D^{\sim 2}}{2c\varepsilon_0}\lambda_1^{-2}\lambda_2^{-2}, & \lambda_1\lambda_2 > s. \end{cases} \tag{4}$$

Also, from Eq. (4), the Hessian matrix of an electromechanical coupling system in Eq. (2) can be expressed as

$$H = \begin{cases} \begin{bmatrix} U_{11} + \dfrac{D^{\sim 2}}{\varepsilon_0}(3\lambda_1^{-3}\lambda_2^{-1}M_1 + a\lambda_1^{-2}N_1) & U_{12} + \dfrac{D^{\sim 2}}{2\varepsilon_0}(\lambda_1^{-2}\lambda_2^{-2}P_1 + 2a\lambda_1^{-1}\lambda_2^{-1}N_1) & -\dfrac{D^{\sim}}{\varepsilon_0}Q_1 \\ U_{12} + \dfrac{D^{\sim 2}}{2\varepsilon_0}(\lambda_1^{-2}\lambda_2^{-2}P_1 + 2\lambda_1^{-1}\lambda_2^{-1}N_1) & U_{22} + \dfrac{D^{\sim 2}}{\varepsilon_0}(3\lambda_1^{-1}\lambda_2^{-3}M_1 + a\lambda_2^{-2}N_1) & -\dfrac{D^{\sim}}{\varepsilon_0}L_1 \\ -\dfrac{D^{\sim}}{\varepsilon_0}Q_1 & -\dfrac{D^{\sim}}{\varepsilon_0}L_1 & \dfrac{\lambda_1^{-2}\lambda_2^{-2}}{\varepsilon_0}K_1 \end{bmatrix}, & \lambda_1\lambda_2 \le s, \\ \begin{bmatrix} U_{11} + \dfrac{3D^{\sim 2}}{c\varepsilon_0}\lambda_1^{-4}\lambda_2^{-2} & U_{12} + \dfrac{2D^{\sim 2}}{c\varepsilon_0}\lambda_1^{-3}\lambda_2^{-3} & -\dfrac{2D^{\sim}}{c\varepsilon_0}\lambda_1^{-3}\lambda_2^{-2} \\ U_{12} + \dfrac{2D^{\sim 2}}{c\varepsilon_0}\lambda_1^{-3}\lambda_2^{-3} & U_{22} + \dfrac{3D^{\sim 2}}{c\varepsilon_0}c\lambda_2^{-4}\lambda_1^{-2} & -\dfrac{2D^{\sim}}{c\varepsilon_0}\lambda_2^{-3}\lambda_1^{-2} \\ -\dfrac{2D^{\sim}}{c\varepsilon_0}\lambda_1^{-3}\lambda_2^{-2} & -\dfrac{2D^{\sim}}{c\varepsilon_0}\lambda_2^{-3}\lambda_1^{-2} & \dfrac{1}{c\varepsilon_0}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix}, & \lambda_1\lambda_2 > s, \end{cases} \tag{5}$$

where

$$M_1 = \frac{(a+b\lambda_1^{-1}\lambda_2^{-1})}{(a\lambda_1\lambda_2+b)^2}, \quad N_1 = \frac{(3a+2b\lambda_1^{-1}\lambda_2^{-1})}{(a\lambda_1\lambda_2+b)^3}, \quad P_1 = \frac{(3a+4b\lambda_1^{-1}\lambda_2^{-2})}{(a\lambda_1\lambda_2+b)^2},$$

$$Q_1 = \frac{3a\lambda_1^{-2}\lambda_2^{-1} + 2b\lambda_1^{-3}\lambda_2^{-2}}{(a\lambda_1\lambda_2+b)^2}, \quad L_1 = \frac{3a\lambda_1^{-1}\lambda_2^{-2} + 2b\lambda_1^{-2}\lambda_2^{-3}}{(a\lambda_1\lambda_2+b)^2}, \quad K_1 = \frac{1}{a\lambda_1\lambda_2+b}. \tag{6}$$

In the electromechanical coupling system of a dielectric elastomer, the relation between nominal electric field $E^{\sim}$ and nominal electric displacement field $D^{\sim}$ can be expressed as $E^{\sim} = D^{\sim}\lambda_1^{-2}\lambda_2^{-2}/\varepsilon$. Therefore, $D^{\sim}$ is $D^{\sim} = E^{\sim}\varepsilon\lambda_1^2\lambda_2^2$ and the real electric field can be expressed as $E = D^{\sim}/(\lambda_1\lambda_2\varepsilon)$.

According to Eq. (5), the determinant in the two stretch ranges can be calculated. When $\det(H) = 0$, the original stable system gets meta-stable because of the electrical breakdown of material. In this critical condition, the relationship between critical real electric field and stretch ratio is as follows:

$$\varepsilon_0 E_{max}^2 = \begin{cases} \dfrac{-B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1}, & \lambda_1\lambda_2 \le s, \\ \dfrac{1}{6c}[(4U_{12}\lambda_1\lambda_2 - U_{11}\lambda_1^2 - U_{22}\lambda_2^2) + \sqrt{(4U_{12}\lambda_1\lambda_2 - U_{11}\lambda_1^2 - U_{22}\lambda_2^2)^2 + 12\lambda_1^2\lambda_2^2(U_{11}U_{22} - U_{12}^2)}, & \lambda_1\lambda_2 > s, \end{cases} \tag{7}$$

where

$$A_1 = (9M_1^2K_1 + 6M_1K_1N_1\lambda_1\lambda_2 a - 3M_1L_1^2\lambda_1^3\lambda_2^5 - aN_1\lambda_1^4\lambda_2^6L_1^2 - P_1K_1N_1\lambda_1\lambda_2 a + P_1Q_1L_1\lambda_1^4\lambda_2^4 + 2N_1Q_1L_1\lambda_1^5\lambda_2^5 a - 3Q_1^2M_1\lambda_1^5\lambda_2^3 - Q_1^2N_1\lambda_1^6\lambda_2^4 a - 0.25P_1^2K_1)(a\lambda_1\lambda_2+b)^4,$$

$$B_1 = (3U_{11}K_1M_1\lambda_1\lambda_2^{-1} + U_{11}K_1N_1\lambda_1^2 a - U_{11}\lambda_1^4\lambda_2^4L_1^2 + 3U_{22}K_1M_1\lambda_1^{-1}\lambda_2 + U_{22}K_1N_1\lambda_2^2 a - 2U_{12}K_1N_1\lambda_1\lambda_2 a + 2U_{12}Q_1L_1\lambda_1^4\lambda_2^4 - U_{22}Q_1^2\lambda_1^4\lambda_2^4)(a\lambda_1\lambda_2+b)^2,$$

$$C_1 = (U_{11}U_{22} - U_{12}^2K_1). \tag{8}$$

The critical nominal electric field $E_{max}^{\sim}$ of a dielectric elastomer can be expressed as follows:

$$\varepsilon_0 E_{max}^{\sim 2} = \begin{cases} \dfrac{-B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1\lambda_1^2\lambda_2^2}, & \lambda_1\lambda_2 \le s, \\ \dfrac{1}{6c}[(4U_{12}\lambda_1^{-1}\lambda_2^{-1} - U_{11}\lambda_2^{-2} - U_{22}\lambda_1^{-2}) + \sqrt{(4U_{12}\lambda_1^{-1}\lambda_2^{-1} - U_{11}\lambda_2^{-2} - U_{22}\lambda_1^{-2})^2 + 12\lambda_1^{-2}\lambda_2^{-2}(U_{11}U_{22} - U_{12}^2)}, & \lambda_1\lambda_2 > s, \end{cases} \tag{9}$$

$$s_j = \frac{\partial W}{\partial \lambda_j} = \begin{cases} U_j - \lambda_j^{-1}\varepsilon_0 E_{max}^2, & \lambda_1\lambda_2 \le s, \\ U_j - \lambda_j^{-1}\varepsilon_0 E_{max}^2, & \lambda_1\lambda_2 > s, \end{cases} \tag{10}$$

where $U_j = \partial U/\partial\lambda_j$, $j=1,2$. Equations (7), (9), and (10) are the generic stability parameters of the electromechanical coupling system of a dielectric elastomer.

A special case of equal biaxial stretch with two principal stretch ratios $\lambda_1=\lambda_2=\lambda$ is taken into consideration as shown below. $U_{11}=U_{22}$ holds under this condition.

From Eqs. (7), (9), and (10),

$$\varepsilon_0 E_{max}^2 = \begin{cases} \dfrac{-B+\sqrt{B^2-4AC}}{2A}, & \lambda_1\lambda_2 \le s, \\ \dfrac{\lambda^2}{3c}(U_{11}+U_{12}), & \lambda_1\lambda_2 > s, \end{cases} \tag{11}$$

$$\varepsilon_0 \tilde{E}_{max}^2 = \begin{cases} \dfrac{-B+\sqrt{B^2-4AC}}{2A\lambda^4}, & \lambda_1\lambda_2 \le s, \\ \dfrac{\lambda^{-2}}{3c}(U_{11}+U_{12}), & \lambda_1\lambda_2 > s, \end{cases} \tag{12}$$

$$s = \begin{cases} U_1 - \dfrac{-B+\sqrt{B^2-4AC}}{2A\lambda}, & \lambda_1\lambda_2 \le s, \\ U_1 - \dfrac{\lambda}{3c}(U_{11}+U_{12}), & \lambda_1\lambda_2 > s. \end{cases} \tag{13}$$

Accordingly, the idiographic expressions of A, B, C, M, N, P, Q, and K given in Eqs. (6) and (8), which are simplified under the condition of $\lambda_1=\lambda_2=\lambda$ and $U_{11}=U_{22}$. Equations (11)–(13) are the simplified expressions of the stability parameters in the special case of equal biaxial stretch with $\lambda_1=\lambda_2=\lambda$. Note that no specific elastic strain energy function is assumed for all the results above.

As an example, free energy function is established to analyze the electromechanical stability performance of a dielectric elastomer by applying neo-Hookean elastic strain energy model in the following section:

$$W(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},\tilde{D}) = \begin{cases} \dfrac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3) + \dfrac{\tilde{D}^2}{2\varepsilon_0(a\lambda_1\lambda_2+b)}\lambda_1^{-2}\lambda_2^{-2}, & \lambda_1\lambda_2 \le s, \\ \dfrac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3) + \dfrac{\tilde{D}^2}{2c\varepsilon_0}\lambda_1^{-2}\lambda_2^{-2}, & \lambda_1\lambda_2 > s, \end{cases} \tag{14}$$

If $\lambda_1\lambda_2 \le s$, the stability parameters of a dielectric elastometer can be written by Eqs. (11)–(13). According to the above mentioned definition of U_{ij}, in this case, U_{11} and U_{22} in Eq. (13) are $\mu(1+3\lambda^{-6})$ and $2\mu\lambda^{-6}$, respectively. After simplification, it is evident that the nominal and the real electric field reach their critical values when the nominal stress becomes zero, i.e., $s=0$. This condition gives critical stretch ratio λ_{max}. Substituting λ_{max} into Eqs. (12) and (11), the critical nominal and the real electric field can therefore be evaluated.

Similarly, if $\lambda_1\lambda_2 > s$,

$$c\varepsilon_0 \tilde{E}_{max}^2 = \frac{\mu}{3}(\lambda^{-2}+5\lambda^{-8}), \tag{15}$$

$$c\varepsilon_0 E_{max}^2 = \frac{\mu}{3}(\lambda^{2}+5\lambda^{-4}), \tag{16}$$

$$s = \frac{2\mu}{3}(\lambda - 4\lambda^{-5}). \tag{17}$$

By taking $s=0$ in Eq. (17), stretch ratio λ reaches its critical value $\lambda_{max}=1.26$. Accordingly, from Eqs. (15) and (16), $\tilde{E}_{max}=0.69\sqrt{\mu/c\varepsilon_0}$, $E_{max}=1.09\sqrt{\mu/c\varepsilon_0}$. These results coincide with the conclusions by Zhao and Suo.[2]

To sum up, the stability parameters of a dielectric elastomer, such as critical nominal electric field, critical real electric field, and nominal stress, are studied as the functions of principal stretch ratios in this work. The accurate expressions of these parameters are presented for a special case of equal biaxial stretch. The results obtained by utilizing the single material constant neo-Hookean model of the elastic strain energy function coincide with the conclusions by Zhao and Suo. We believe that the appropriate elastic strain energy function of a specific elastomer material can be used to predict the stability of material. The results presented in this work can be used to guide the design and fabrication of dielectric elastomer actuators.

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