# Constitutive model for shape memory alloy torsion actuator

ZHOU Bo[1,2], LENG Jin-song[2], ZOU Guang-ping[1], LIU Yan-ju[3], ZHOU Li-min[4]

周 博, 冷劲松, 邹广平, 刘彦菊, 周利民

(1. College of Civil Engineering, Harbin Engineering University, Harbin 150001, China, zhoubo@hrbeu.edu.cn;
2. Center for Composite Materials and Structures, Harbin Institute of Technology, Harbin 150080, China;
3. Dept. of Aerospace Science and Mechanics, Harbin Institute of Technology, Harbin 150001, China;
4. Dept. of Mechanical Engineering, The Hong Kong Polytechnic University, Hong Kong, China)

**Abstract**: Shape memory alloy (SMA) torsion actuator is one of the key approaches realizing adaptive wings in airplanes. In this paper, the actuator is made up of SMA wires and a thin-walled tube, in which the SMA wires are twisted and affixed around the surface of the tube at an angle referenced to the center axis of the tube. A thermo-mechanical constitutive model is developed to predict the thermo-mechanical behaviors of the SMA torsion actuator based on the knowledge of solid mechanics. The relationship between the torsion-angle and temperature is numerically calculated by using the thermo-mechanical constitutive model coupled with the SMA phase transformation model developed by Zhou and Yoon. The numerical results are compared with the relative experimental results finished by Xiong and Shen. Influences of the twist-angle of SMA wires and geometrical factors on the primary actuation performances of the SMA torsion actuator are also numerically investigated based on the thermo-mechanical constitutive model coupled with the SMA phase transformation model developed by Zhou and Yoon. Results show that the thermo-mechanical constitutive model can well predict the thermo-mechanical behaviors of the SMA torsion actuator.

**Key words**: shape memory alloy; torsion actuator; thermo-mechanical constitutive model
**CLC number**: V259 **Document code**: A **Article ID**: 1005-9113(2010)02-0278-05

Shape memory alloy (SMA) is a kind of smart material with special thermo-mechanical properties associated with super elasticity and shape memory effect. Both special properties are the results of the reversible solid-state transformations between austenitic and martensitic phases. Compared with the other smart materials, such as materials of piezoelectricity, magnetostriction and so on, SMA can produce larger recovery strain or stress, so they become currently interesting candidates used to design variety of actuators and sensors. Ni – Ti SMA has been being widely studied and used in different engineering fields, for it can not only produce larger recovery strain (or/and stress) and greater work per unit weight but also have stable thermo-mechanical behavior after a long period of reversible activation[1-3].

Adaptive wings can improve the aerodynamic performance of airplanes significantly. There are actuators assembled by smart materials and ordinary structural materials in adaptive wings. The shape and spanwise angle of adaptive wings can be changed in elastic range through the active character of smart materials in the actuator to obtain the best aerodynamic behavior. The adaptive wings possess simpler surface structure and higher fatigue strength than the ordinary wings. They also have the potential capacity and predominance to reduce the weight and critical load, and to improve the agility and radar radiation-section of airplanes. SMA torsion actuator is one of the key approaches realizing adaptive wings in airplanes [4-6].

In this study, a SMA torsion actuator is made up of SMA wires and a thin-walled tube of ordinary metal. In the actuator, the inelastic elongated wires are twisted and affixed around the surface of the tube at a certain angle referenced to the center axis of the tube. The change of temperature produces recovery stress in SMA wires, which can manage the wing spanwise torsion. A thermo-mechanical constitutive model is established to predict the actuation performances of the SMA torsion by using the related knowledge of solid mechanics. The relations of torsion-angle versus temperature during the heating process are simulated by the thermo-mechanical constitutive model coupled with the SMA phase transformation model established by Zhou and Yoon[7]. The results of the numerical simulations are compared with the previous relative experimental results performed by Xiong and Shen[3]. The influences of the twist-angle of SMA wires and the geometrical dimensions on the thermo-mechanical behaviors of the SMA torsion actuator are

Received 2008 – 03 – 07.
Sponsored by the Postdoctoral Science Foundation of China (Grant No. 20080430933) and the National Natural Science Foundation of China (Grant No. 90505010).

also numerically researched in detail base on the thermo-mechanical constitutive model for the SMA torsion.

## 1 Structure of SMA Torsion Actuator

The components of a SMA torsion actuator include SMA wires and a thin-walled tube[2-3] (as shown in Fig. 1). Several SMA wires with pre-elongated deformations are twisted and affixed around the surface of the thin-walled metal tube at an angle, $\alpha$, referenced to the center axis, $X$, of the tube. The length, external diameter and wall thickness of the tube are denoted by $L$, $D$, and $t$ respectively. In this study, the angle, $\alpha$, is called as the twist-angle of SMA wires. The length and number of SMA wires in the SMA torsion actuator are denoted by $l$ and $N$ respectively. It should be emphasized that only one SMA wire is drawn in the structural diagram of the SMA torsion actuator (as shown in Fig. 1), which make it clearer to illustrate the structure of the SMA torsion actuator.

The length of SMA wires, $l$, can be expressed as the following function of the tube length, $L$, and the twist-angle of SMA wires, $\alpha$.

$$l = \frac{L}{\cos \alpha}. \qquad (1)$$

The number of SMA wires, $N$, diameter of SMA wires, $d$, and the external diameter of thin-walled tube, $D$, should satisfy the following inequation.

$$N \leqslant \frac{\pi D}{d} \cos \alpha. \qquad (2)$$

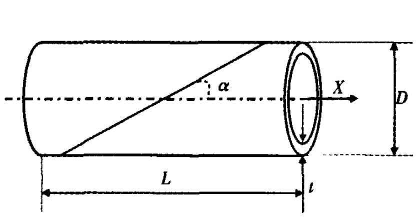


**Fig. 1 Structural diagram of SMA torsion actuator[2]**

The SMA torsion actuator does not produce bending deformation during its normal operations, which is due to that the SMA wires are symmetrically assembled in the torsion actuator. It will only have the deformations of compression and torsion accordingly. In order to simplify the theoretical derivations, the following assumptions are made: 1) The SMA wires only produce deformation along axis-direction and have no bending deformation, which is because the SMA wires are so slender that their bending deformation is small enough to be ignored. 2) Both distributions of normal and shear stresses in the cross-section of the thin-walled metal tube are uniform. 3) The SMA wires and the thin-walled tube are perfectly affixed. There is no opposite slippage occurring between the SMA wires and the thin-walled tube during the operation of the SMA torsion actuator. 4) The changes of temperature have no influence on the thermo-mechanical behaviors of the thin-wall metal tube. This is because the influence of temperature changing on the mechanical behaviors of thin-walled tube is far less than that on the thermo-mechanical behaviors of SMA wires.

## 2 Driving Stress Equation

For SMA in one dimensional stress-state, the relationship of stress, $\sigma$, strain, $\varepsilon$, martensitic volume fraction, $\xi$, and temperature, $T$, is expressed as[8]

$$\sigma - \sigma_0 = E(\varepsilon - \varepsilon_0) + \Omega(\xi - \xi_0) + \Theta(T - T_0). \qquad (3)$$

where $E$, $\Omega$ and $\Theta$ are elastic modulus, phase transformation modulus and thermo-expansion modulus of SMA respectively. $\sigma_o$, $\varepsilon_o$, $\xi_o$ and $T_o$ are the initial values of $\sigma$, $\varepsilon$, $\xi$ and $T$ respectively. There is a functional relationship between the phase transformation modulus, $\Omega$, and elastic modulus, $E$, expressed as[8]

$$\Omega = -\varepsilon_L E. \qquad (4)$$

where $\varepsilon_L$ is a material constant of SMA, called as material maximal residual strain, which can be determined through tensile test. Substituting Eq. (4) into Eq. (3) leads to

$$\varepsilon_0 - \varepsilon = \frac{\sigma_0 - \sigma}{E} + \varepsilon_L(\xi_0 - \xi) + \frac{\Theta}{E}(T - T_0). \qquad (5)$$

Martensitic volume fraction, $\xi$, is the function of the material stress, $\sigma$, and material temperature, $T$, during the transformations between martensitic and austenitic phases, which is expressed by

$$\xi = \xi(T, \sigma). \qquad (6)$$

Phase transformation models describing the functional relationship, Eq. (6), play an important role in studying the thermo-mechanical behaviors of structures with SMA. In 1986, Tanaka[9] developed an exponent-type phase transformation model based the nucleation kinetics differential equation for metal materials. In 1990, Liang and Rogers[8] found that Tanaka's model can not accord with the experimental results for some SMAs and suggested a cosine-type model based on the experimental results. Above two SMA phase transformation models are currently used in various practical applications due to their simple mathematical expressions. In 2006, Zhou and Yoon[7] developed a triangle-type phase transformation model from the differential relation between the martensitic volume fraction and free energy during the phase transformation of SMA. Differently form the models of Tanaka's and Liang's, this model is formulated on the phase transformation peak temperature as well as the phase transformation starting and finishing temperatures, which make it predict the phase transformation behaviors of SMA more precisely.

During the heating process, the shortened distance of SMA wires and their projections along the axis direction of tube are expressed as $\Delta l$ and $\Delta L$ respectively. The functional relationship between them is

$$\Delta l \cos \alpha = \Delta L. \quad (7)$$

If the normal stress and elastic modulus of the thin-walled tube are denoted by $\sigma_t$ and $E_t$ respectively, Eq. (8) can be obtained from Eq. (7) using the knowledge of solid mechanics.

$$l \times (\varepsilon_0 - \varepsilon)\cos \alpha = \frac{\sigma_t}{E_t} L. \quad (8)$$

Substituting Eqs. (1) and (5) into Eq. (8) leads to

$$\frac{\sigma_t}{E_t} = \frac{\sigma_0 - \sigma}{E} + \varepsilon_L(\xi_0 - \xi) + \frac{\Theta}{E}(T - T_0). \quad (9)$$

During its operation, the equilibrium equation of SMA torsion actuator in axis direction, $X$, is

$$N\frac{\pi}{4}d^2\sigma\cos \alpha - \pi Dt\sigma_t = 0. \quad (10)$$

where $d$ is the diameter of SMA wire. According to above equation

$$\sigma_t = \frac{Nd^2\cos \alpha}{4Dt}\sigma. \quad (11)$$

Using Eqs. (9) and (11) simultaneously leads to the driving stress equation of SMA wires in the SMA torsion actuator, expressed as

$$\sigma = \beta[\sigma_0 + E\varepsilon_L(\xi_0 - \xi) + \Theta(T - T_0)]. \quad (12a)$$

where

$$\beta = \frac{E_t}{E\alpha_c + E_t}, \quad (12b)$$

$$\alpha_c = \frac{Nd^2\cos \alpha}{4Dt}. \quad (12c)$$

The coefficient, $\alpha_c$, in Eq. (12c) and quantity, $\beta$, in Eq. (12b) describes the geometrical and physical characters of the SMA torsion actuator respectively. During the phase conversion of SMA from martensite to austenite, the relationships between critical stress and critical temperature read as the following two equations[8].

$$\sigma_s = C_A(A_s - A_{so}). \quad (13a)$$

and

$$\sigma_f = C_A(A_f - A_{fo}). \quad (13b)$$

where $\sigma_s$ and $\sigma_f$ are phase conversion starting and finishing stresses respectively. $A_s$ and $A_f$ are phase starting and finishing temperatures respectively. $A_{so}$ and $A_{fo}$ are the values of $A_s$ and $A_f$ in the state of free-stress respectively. $C_A$ is a material constant describing the relationship between the phase conversion critical stress and temperature of SMA.

When the phase conversion from martensite to austenite starts, martensite volume fraction $\xi = \xi_o$. According to Eq. (12a), the phase conversion starting stress can be expressed as

$$\sigma_s = \beta[\sigma_0 + \Theta(A_s - T_0)]. \quad (14a)$$

When the phase conversion from martensite to austenite finishes, martensite volume fraction $\xi = 0$. According to Eq. (12a), the phase conversion finishing stress can be expressed as

$$\sigma_f = \beta[\sigma_0 + E\varepsilon_L\xi_0 + \Theta(A_f - T_0)]. \quad (14b)$$

Substituting Eq. (13a) into Eq. (14a) and Eq. (13b) into (14b) lead to the formulas of critical temperatures of SMA torsion actuator, expressed as

$$A_s = \frac{C_A A_{so} - \beta\Theta T_0 + \beta\sigma_0}{C_A - \beta\Theta}. \quad (15a)$$

and

$$A_f = \frac{C_A A_{fo} - \beta\Theta T_0 + \beta\sigma_0 + \beta E\varepsilon_L\xi_0}{C_A - \beta\Theta}. \quad (15b)$$

respectively.

Substituting Eq. (15a) into Eq. (13a) and Eq. (15b) into Eq. (13b), the formulas of critical stress of SMA torsion actuator are easily expressed as

$$\sigma_s = C_A\beta\frac{\Theta(A_{so} - T_0) + \sigma_0}{C_A - \beta\Theta}. \quad (16a)$$

and

$$\sigma_f = C_A\beta\frac{\Theta(A_{fo} - T_0) + \sigma_0 + E\varepsilon_L\xi_0}{C_A - \beta\Theta}. \quad (16b)$$

respectively.

## 3 Torsion-angle Equation

The moment equilibrium around the center axis of the torsion actuator, shown in Fig. 1, requires

$$\pi Dt\tau_t\frac{D}{2} - N\frac{\pi d^2}{4}\sigma\frac{D}{2}\sin \alpha = 0. \quad (17)$$

where $\tau_t$ is the shear stress in the cross-section of the thin-walled tube and $\sigma$ is the normal stress of SMA wires. From above equation, the shear stress $\tau_t$ is expressed as

$$\tau_t = \alpha_s\sigma. \quad (18a)$$

where

$$\alpha_s = \frac{Nd^2\sin \alpha}{4Dt}. \quad (18b)$$

According to the shear Hook's law,

$$\gamma_t = \frac{\tau_t}{G_t}. \quad (19)$$

where $G_t$ and $\gamma_t$ are the shear modulus and shear strain in the cross-section of thin-walled tube respectively. The torsion angle between both ends of the torsion actuator, shown in Fig. 1, is expressed as a function of the shear strain in the cross-section of thin-walled tube as

$$\phi = \frac{L\gamma_t}{D/2}. \quad (20)$$

Using Eqs. (12a), (18a), (19) and (20) simultaneously leads to the torsion-angle equation of the SMA torsion actuator, expressed as

$$\phi = \frac{2L\alpha_s\beta}{DG_t}[\sigma_0 + E\varepsilon_L(\xi_0 - \xi) + \Theta(T - T_0)]. \tag{21}$$

The actuation performances of the SMA torsion actuator can be predicted by using this equation coupled with the phase transformation model of Tanaka's[9], Liang's[8] or Zhou's[7].

## 4 Numerical Examples

The thermo-mechanical behaviors of the SMA torsion actuator are numerically simulated by using the torsion-angle equation, Eq. (21) coupled with Zhou's SMA phase transformation model[7]. During the numerical simulations, the material constants and geometrical dimensions of SMA wires and thin-walled tube in the torsion actuator are taken from Refs. [2] and [8], listed in Tab. 1.

**Tab. 1 Material constants and geometric dimensions**

| Material constants and geometric dimensions of SMA wires | | | | |
|---|---|---|---|---|
| $E$/GPa | $\Theta$/(MPa · ℃$^{-1}$) | $C_A$/(MPa · ℃$^{-1}$) | $\varepsilon_L$ | |
| 40 | 0.55 | 14 | 0.07 | |
| $A_{so}$/℃ | $A_{po}$/℃ | $A_{fo}$/℃ | $d$/mm | |
| 35 | 40 | 50 | 0.5 | |
| **Material constants and geometric dimensions of thin-walled tube** | | | | |
| $E_t$/GPa | $G_t$/GPa | $D$/mm | $t$/mm | $L$/mm |
| 120 | 40 | 20 | 0.5 | 50 |

Fig. 2 shows the relationship curves of torsion-angle versus temperature in the SMA torsion actuator with different twist-angles of 40°, 45°, 50°, 55° and 60°. During the numerical calculations, the number of SMA wires is $N$ = 90. It is well known that the torsion-angle significantly increases with the increasing of temperature during the temperature region from 40 ℃ to 120 ℃, which is the results of the phase transform from martensite to austenite in the SMA wires. During the process of phase transformation, the relationship between torsion-angle and temperature is approximately linear, which accords with the experimental results finished by Xiong and Shen[3]. The curves in this figure indicate the maximum torsion-angle, which corresponds with the point of phase transformation finishing, increases when the twist-angle of SMA wires become larger. This also accords with the experimental results finished by Xiong and Shen[3]. So the actuation effects of the SMA torsion actuator can be enhanced through increasing the twist angle of SMA wires.

Fig. 3 shows the curve of maximum torsion-angle versus twist-angle. It is well known that the maximum torsion-angle nonlinearly increases with the increasing of the twist-angle of SMA wires. It should be emphasized that if the value of the twist angle of SMA wires, $\alpha$, approaches to zero, the torsion shear stress of thin-walled tube induced by the driving stress of SMA wires will approach to zero and there will be no torsion angle as expected occurring in the SMA torsion actuator, while if the value of the twist angle of SMA wires, $\alpha$, approaches to 90°, it is impossible to satisfy Eq. (2). So the reasonable region of twist-angle of the SMA wires is from 40° to 60°, which accords with the experimental results finished by Xiong and Shen[3].

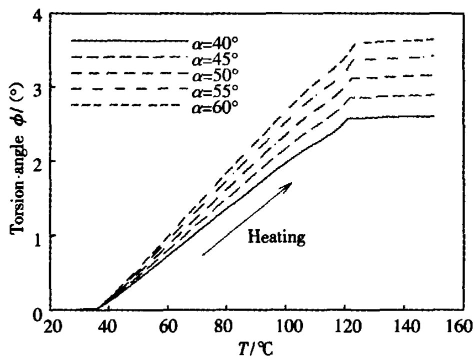


**Fig. 2 Curves of torsion-angle versus temperature with various twist-angle**

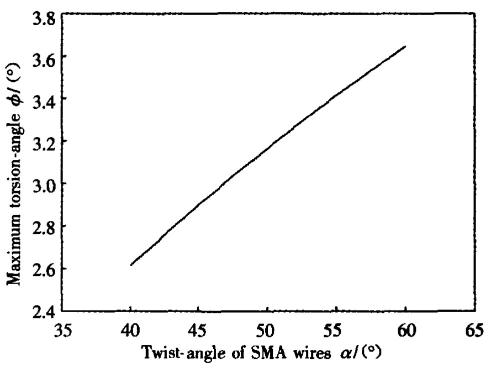


**Fig. 3 Curve of maximum torsion-angle versus twist-angle**

Fig. 4 shows curves of torsion-angle versus temperature with various SMA wire number of 50, 60, 70, 80 and 90. The twist-angle of SMA wires is $\alpha$ = 45°. Similarly with Fig. 2, the torsion-angle obviously increases with the increasing of temperature during the temperature region from 40 ℃ to 120 ℃, which is due to the phase transform from martensite to austenite in the SMA wires. During the process of phase transformation, the relationship between torsion-angle and temperature is approximately linear, which also accords with the experimental results finished by Xiong and Shen[3]. It is well known that the torsion-angle of the SMA torsion actuator increases with the increasing of number of SMA wires.

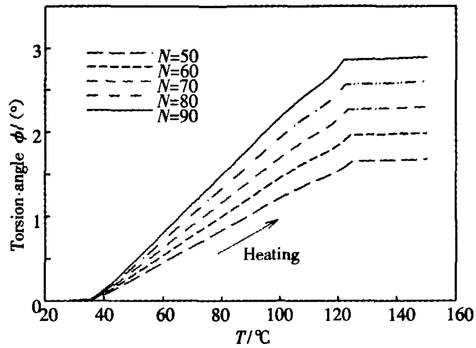


**Fig. 4 Curves of torsion-angle versus temperature with various SMA wire number**

Fig. 5 shows the curve of maximum torsion-angle versus the number of SMA wires. It is well known that the maximum torsion-angle linearly increases with the increasing of the number of SMA wires, which accords with Eqs. (18b) and (21). The actuation effects of the SMA torsion actuator can be enhanced through increasing the number of SMA wires in the torsion actuator. But the number of SMA wires should satisfy both Eq. (1) and Inequation (2).

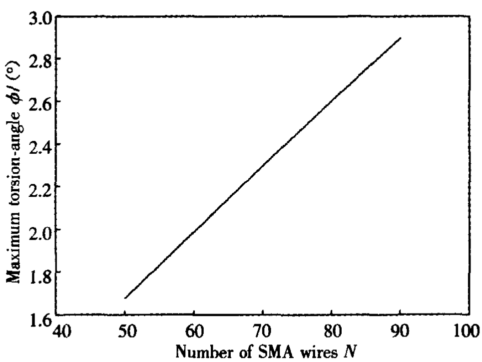


**Fig. 5 Curve of maximum torsion-angle versus the number of SMA wires**

## 5 Conclusions

A thermo-mechanical constitutive model for SMA torsion actuator, which is made of thin-walled metal tube and SMA wires, is developed based on Liang's SMA thermo-mechanical equation and the relative knowledge of solid mechanics. The actuation performances of the SMA torsion actuator are numerically simulated by this thermo-mechanical constitutive model coupled with the phase transformation model established by Zhou and Yoon. The influences of the geometrical factors on the actuation performances of the SMA torsion actuator are also numerically investigated by using this thermo-mechanical constitutive model coupled with the phase transformation model established by Zhou and Yoon.

Numerical simulations show that the torsion-angle of the SMA torsion actuator significantly increases with the increasing of temperature during the phase transform from martensite to austenite in the SMA wires. During the process of phase transformation upon heating, the relationship between torsion-angle and temperature is approximately linear. The value of the maximum torsion-angle of the SMA torsion actuator nonlinearly increases with the increasing of the value of the twist-angle of SMA wires. However the value of the maximum torsion-angle of the SMA torsion actuator linearly increases with the increasing of the number of SMA wires. The comparisons between the numerical simulations and the experimental results finished by Xiong and Shen illustrate that the developed thermo-mechanical constitutive is able to predict the thermo-mechanical behaviors of the SMA torsion actuator competently.

## References:

[1] Zhao L C, Cai W, Zheng Y F. Shape Memory Effect and Superelasticity in Alloys. Beijing: National Defence Industry Press, 2002. 1 - 66.

[2] Xiong K, Tao B Q, Yao E T. Mechanical model and experimental analyses on shape memory alloy torsion actuator. Acta Aeronautica Et Astronautica Sinica, 2001, 22(7): 379 - 381.

[3] Xiong K, Shen W. Experimental research on the relationship between SMA wire wound angle and SMA torsion actuator primary performances. Chinese Journal of Mechanical Engineering, 2003, 39(12): 123 - 128.

[4] Davidson F M, Liang C. Investigation of torsional shape memory alloys actuators. Proceedings of the Conference on Smart Structures and Materials, SPIE, 1996, 2717: 672 - 682.

[5] Jardine A P, Jonathan B C, Flanagan J. Improved design and performance of the SMA torque tube for DARPA smart wing program. Proceedings of the conference on Industrial and Commercial Applications of Smart Structures Technologies, SPIE, 1999, 3674: 260 - 269.

[6] Keefe A C, Carman G P. Thermo-mechanical characterization of shape memory torque tubes. Proceedings of the conference on Smart Structures and Materials, SPIE, 1999, 3675: 295 - 302.

[7] Zhou B, Yoon S H. A new phase transformation constitutive model of shape memory alloys. Smart Materials and Structures, 2006, 15: 1967 - 1973.

[8] Liang C, Rogers C A. One-dimensional thermomechanical constitutive relations for shape memory materials. Journal of Intelligent Systems and Structures, 1990, 1: 207 - 234.

[9] Tanaka K. A thermomechanical sketch of shape memory effect: one-dimensional tensile behavior. Res Mechanica, 1986, 18: 251 - 263.

# Constitutive model for shape memory alloy torsion actuator

作者： ZHOU Bo， LENG Jin-song， ZOU Guang-ping， LIU Yan-ju， ZHOU Li-min
作者单位： ZHOU Bo(College of Civil Engineering, Harbin Engineering University, Harbin 150001, China;Center for Composite Materials and Structures, Harbin Institute of Technology, Harbin 150080, China)， LENG Jin-song(Center for Composite Materials and Structures, Harbin Institute of Technology, Harbin 150080, China)， ZOU Guang-ping(College of Civil Engineering, Harbin Engineering University, Harbin 150001, China)， LIU Yan-ju(Dept. of Aerospace Science and Mechanics, Harbin Institute of Technology, Harbin 150001, China)， ZHOU Li-min(Dept. of Mechanical Engineering, The Hong Kong Polytechnic University, Hong Kong, China)
刊名： 哈尔滨工业大学学报（英文版）EI
英文刊名： JOURNAL OF HARBIN INSTITUTE OF TECHNOLOGY
年，卷(期)： 2010,17(2)

## 参考文献(9条)

1. Tanaka K A thermomechanical sketch of shape memory effect:one-dimensional tensile behavior 1986

2. Liang C;Rogers C A One-dimensional thermomechanical constitutive relations for shape memory materials[外文期刊] 1990

3. Zhou B;Yoon S H A new phase transformation constitutive model of shape memory alloys[外文期刊] 2006(6)

4. Keefe A C;Carman G P Thermo-mechanical characterization of shape memory torque tubes 1999

5. Jardine A P;Jonathan B C;Flanagan J Improved design and performance of the SMA torque tube for DARPA smart wing program 1999

6. Davidson F M;Liang C Investigation of torsional shape memory alloys actuators 1996

7. Xiong K;Shen W Experimental research on the relationship between SMA wire wound angle and SMA torsion actuator primary performances[期刊论文]-Chinese Journal of Mechanical Engineering 2003(12)

8. Xiong K;Tao B Q;Yao E T Mechanical model and experimental analyses on shape memory alloy torsion actuator 2001(07)

9. Zhao L C;Cai W;Zheng Y F Shape Memory Effect and Superelasticity in Alloys 2002