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A macro-mechanical constitutive model of shape memory alloys

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It is of practical interest to establish a precise constitutive model which includes the equations describing the phase transformation behaviors and thermo-mechanical processes of shape memory alloy (SMA). The microscopic mechanism of super elasticity and shape memory effect of SMA is explained based on the concept of shape memory factor defined by the author of this paper. The conventional super elasticity and shape memory effect of SMA are further unified as shape memory effect. Shape memory factor is redefined in order to make clear its physical meaning. A new shape memory evolution equation is developed to predict the phase transformation behaviors of SMA based on the differential relationship between martensitic volume fraction and phase transformation free energy and the results of DSC test. It overcomes the limitations that the previous shape memory evolution equations or phase transformation equations fail to express the influences of the phase transformation peak temperatures on the phase transformation behaviors and the transformation from twinned martensite to detwinned martensite occurring in SMA. A new macro-mechanical constitutive equation is established to predict the thermo-mechanical processes realizing the shape memory effect of SMA from the expression of Gibbs free energy. It is expanded from one-dimension to three-dimension with assuming SMA as isotropic material. All material constants in the new constitutive equation can be determined from macroscopic experiments, which makes it more easily used in practical applications.

shape memory alloy, shape memory factor, shape memory evolution equation, constitutive equation

Shape memory alloy (SMA) has been used in many various practical applications due to its unique super elasticity and shape memory effect, excellent physical chemical character and biocompatibility[1]. Researchers from different countries and regions have made a great of progress in establishing the constitutive model of SMA within recent 20 years. The constitutive model of SMA includes the phase transformation equation, which describes the phase transformation behaviors of SMA, and the mechanical constitutive equation, which predicts the thermo-mechanical processes realizing the super elasticity or shape memory effect of SMA. In 1986, Tanaka[2] developed a one-dimensional phase transformation equation of exponential type from the kinetic equation of nucleation for metal materials and established a one-dimensional differential mechanical constitutive equation. In 1990, Liang et al.[3] supposed a one-dimensional phase transformation equation of cosine type and established a one-dimensional mechanical constitutive equation based on the assumption that material parameters of SMA are constants. In 1994, Boyd et al.[4] separated the strain of SMA into three parts of elastic strain, phase transformation strain and thermo-expan-

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ding strain and established a three-dimensional constitutive model of SMA by using the phase transformation equation of exponential type. In 1993, Sun et al.[5] explained the super elasticity and shape memory effect of SMA induced by the non-proportion load on the basis of micro-mechanics, irreversible thermodynamics and the explanation on micro-mechanism and developed a three-dimensional constitutive model by using the phase transformation equation of exponential type. In 1993, Brinson[6] separated martensitic volume fraction into two parts induced by stress and temperature. She supposed a phase transformation equation based on the phase transformation equation of cosine type and developed a one-dimensional mechanical constitutive equation. In 2001, Peng et al.[7] developed a three-dimensional constitutive model according to the dynamic phase transformations induced by stress and temperature and the phase transformation equation of exponential type. In 2001, Zhu et al.[8] established a three-dimensional constitutive model, which formulates the recoverable strain as different expressions during the different processes of phase transformation, by using the phase transformation equation of exponential type. In 2002, Brocca et al.[9] developed a three-dimensional constitutive model on the basis of micro-plane theory and the phase transformation equation of exponential type. In 2004, Li et al.[10] assumed the mechanical behaviors of SMA as the results of dynamic composition of its various phase and developed a three-dimensional constitutive model on the basis of classical plastic theory. In 2003, Guo et al.[11] established a rate dependent phase transformation equation of one-dimension based on the three linear thermo-elastic phase transformation model.

Except Brinson's phase transformation equation, the other phase transformation equations mentioned above fail to describe the transformation from twinned martensite to detwinned martensite occurring in SMA. Brinson's phase transformation equation is able to describe the transformation from twinned martensite to detwinned martensite but its mathematical expression is complex, which make it not very easily used in practical applications. In 2006, Zhou et al.[12] defined the concept of shape memory factor for describing the thermo-mechanical processes of super elasticity and shape memory effect of SMA. They developed a shape memory evolution equation, which can describe the transformation from twinned martensite to detwinned martensite and established a micro-mechanical constitutive equation of three-dimension. In 2006, Zhou et al.[13] developed a phase transformation equation of trigonometric type based on DSC tests and the differential relationship between martensitic volume fraction and phase transformation free energy of SMA. It can express the influences of the phase transformation peak temperatures as well as the phase transformation starting and finishing temperatures on the phase transformation behaviors of SMA. However the other previous phase transformation equations or shape memory evolution equations mentioned above can not express the influences of the phase transformation peak temperatures on the phase transformation behaviors of SMA. The study of Zhou et al.[14] showed the influences of phase transformation peak temperatures are equally important as those of phase transformation starting and finishing temperatures on the phase transformation behaviors of SMA. So the phase transformation equation of trigonometric type can predict the phase transformation behaviors of SMA more precisely. However it also fails to describe the transformation from twinned martensite to detwinned martensite of SMA. In 2008, Zhou et al.[15] established a thermo-mechanical constitutive equation of a torsion actuator of SMA by using the phase transformation equation of trigonometric type. Its numerical results are in good accord with the experimental results conducted by Xiong et al.[16].

In this paper, the microscopic mechanisms of super elasticity and shape memory effect of SMA are illustrated by using the concept of shape memory factor defined by the author of this paper in ref. [12]. The conventional super elasticity and shape memory effect are further unified as shape memory effect. Shape memory factor is redefined to make clear its physical meaning. A new shape memory evolution equation is developed based on the results of DSC test and the differential relationship between martensitic volume fraction and phase transformation free energy of SMA. The new shape memory evolution equation can express both the influences of phase transformation peak temperatures on the phase transformation behaviors and the transformation from the twinned martensite to the detwinned martensite of SMA, which overcomes the limitations of the previous phase transformation equations or shape memory evolution equations of SMA. A new macro-mechanical constitutive equation is established based on the differential relationship between Gibbs free energy and material strain. It is expanded from one-dimension to three-dimension on the basis of assuming SMA as isotropic material. All material constants in the new constitutive equation can be determined through macroscopic experiments, which makes it more easily used in practical applications than the previous three-dimensional constitutive equations of SMA.

1 The shape memory factor

The recoverable non-linear strain of SMA was called shape memory strain and the concept of shape memory factor was defined in ref. [12]. Shape memory factor is a scalar changing between 0 and 1. The thermo-mechanical processes of super elasticity and shape memory effect of SMA can be clearly described with shape memory factor. For super elasticity, shape memory strain increases with the increasing of shape memory factor during the process of loading. During the process of unloading the shape memory strain decreases with the decreasing of shape memory factor. For shape memory effect, the process of loading is the same as that for super elasticity. During the process of unloading both shape memory strain and shape memory factor are unchangeable. After unloading shape memory strain decreases with the decreasing of shape memory factor upon heating. The values of shape memory strain are 0 and the maximum when shape memory factor are 0 and 1 respectively. The relationship between shape memory factor, η, and martensitic volume fraction ξ is

$$\xi = \xi_0 + (1-\xi_0)\eta, \tag{1}$$

where ξ_0 is the initial value of ξ.

The microscopic mechanism of super elasticity and shape memory effect of SMA can be easily described by using shape memory factor. Figure 1(a) shows the microscopic mechanism of super elasticity of SMA. During the process of loading, SMA gradually transforms from austenite to detwinned martensite, and the values of shape memory strain and shape memory factor gradually increase from 0. During the process of unloading, SMA gradually transforms from detwinned martensite to austenite, and the values of shape memory strain and shape memory factor gradually decrease to 0. Figure 1(b) shows the microscopic mechanism of shape memory effect. During the process of loading, SMA gradually transforms from austenite or twinned martensite to detwinned martensite, and the values of shape memory strain and shape memory factor gradually increase from 0. After unloading, SMA gradually transforms from detwinned martensite to austenite, and the values of shape memory strain and shape memory factor gradually decrease to 0 upon heating. It is well-known that both super elasticity and shape memory effect include the process of the transformation to detwinned martensite with the increasing of shape memory factor and the process of the transformation to austenite with the decreasing of shape memory factor. So the conventional super elasticity and shape memory effect of SMA are unified as shape memory effect in this paper. The conventional super elasticity and shape memory effect can be considered as the shape memory effects induced by unloading and temperature respectively. The thermo-mechanical process realizing shape memory effect can be expressed by the changing of shape memory factor. In this paper, shape memory factor is redefined to further make clear its physical meaning. Shape memory factor is an internal invariable describing the thermo-

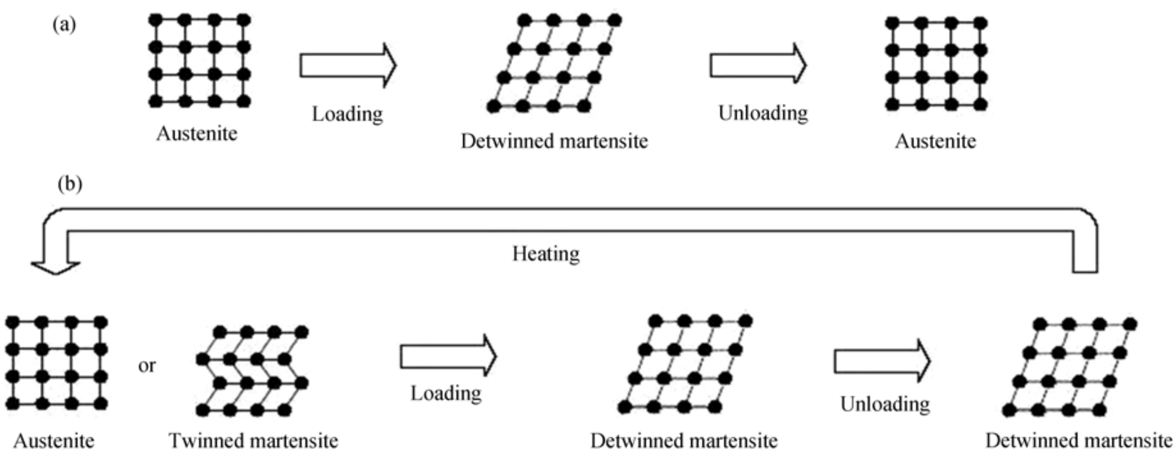

Figure 1 The microscopic mechanism of shape memory effect of SMA. (a) The microscopic mechanism of conventional super elasticity of SMA; (b) the microscopic mechanism of conventional shape memory effect of SMA.

mechanical process realizing shape memory effect of SMA, whose value is between 0 and 1. It is with obvious physical meaning. Shape memory strain gradually comes into being during the increasing of shape memory factor, and gradually recover during the decreasing of shape memory factor. Realizing shape memory effect includes two processes that the value of shape memory factor increases from 0 to 1 and decreases from 1 to 0.

2 Shape memory evolution equation

Figure 2 shows the classical heat flow-temperature curve of SMA from DSC test[14] and its simulative curve in the state of stress free. A_s, A_p and A_f are austenite starting temperature, austenite peak temperature and austenite finishing temperature during the phase transformation from twinned martensite to austenite respectively. M_s, M_p and M_f are martensite starting temperature, martensite peak temperature and martensite finishing temperature during the phase transformation from austenite to twinned martensite respectively. The curve of DSC test reveals that phase transformation starting temperatures (A_s and M_s), phase transformation peak temperatures (A_p and M_p) and phase transformation finishing temperatures (A_f and M_f) are three main factors influencing the phase transformation behaviors of SMA.

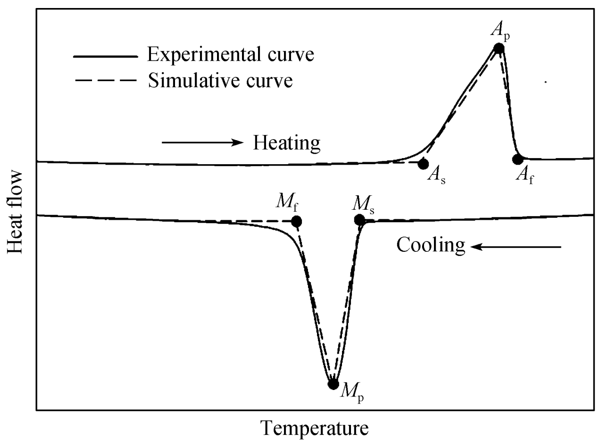

Figure 2 The curve of DSC test and its simulation.

According to the heat flow-temperature simulative curve, the relationship of heat flow and temperature is piecewise linear during the phase transformations between austenite and twinned martensite of SMA. During the phase transformation from twinned martensite to austenite, the function of heat flow-temperature curve is expressed as

$$f(T) = A\frac{T - A_s}{A_p - A_s} \quad (A_s < T < A_p), \tag{2a}$$

$$f(T) = A\frac{T - A_f}{A_p - A_f} \quad (A_p < T < A_f), \tag{2b}$$

where A is a constant. The increment of heat during the phase transformation from twinned martensite to austenite is denoted by $\Delta Q^{M\to A}$. Then we can rewrite eq. (2) as

$$\frac{\mathrm{d}\Delta Q^{M\to A}}{\mathrm{d}T} = A\frac{T - A_s}{A_p - A_s} \quad (A_s < T < A_p), \tag{3a}$$

$$\frac{\mathrm{d}\Delta Q^{M\to A}}{\mathrm{d}T} = A\frac{T - A_f}{A_p - A_f} \quad (A_p < T < A_f). \tag{3b}$$

The increments of free energy and entropy during the phase transformation from twinned martensite to austenite are denoted by $\Delta G^{M\to A}$ and $\Delta S^{M\to A}$ respectively. $\Delta G^{M\to A}$, $\Delta S^{M\to A}$ and $\Delta Q^{M\to A}$ should satisfy

$$\frac{\mathrm{d}\Delta G^{M\to A}}{\mathrm{d}T} = \frac{\mathrm{d}\Delta Q^{M\to A}}{\mathrm{d}T} - \Delta S^{M\to A}, \tag{4}$$

during the phase transformation from twinned martensite to austenite. The differential relationship of martensitic volume fraction and the increment of free energy is

$$\frac{\mathrm{d}\xi}{\mathrm{d}T} = k\frac{\mathrm{d}\Delta G^{M\to A}}{\mathrm{d}T} + B, \tag{5}$$

where k and B are proportional coefficient and a constant, respectively. Using eqs. (3)—(5) and assuming $\mathrm{d}\xi/\mathrm{d}T = 0$ at the temperatures of $T=A_s$ and $T=A_f$, we have

$$\frac{\mathrm{d}\xi}{\mathrm{d}T} = kA\frac{T - A_s}{A_p - A_s} \quad (A_s < T < A_p), \tag{6a}$$

$$\frac{\mathrm{d}\xi}{\mathrm{d}T} = kA\frac{T - A_f}{A_p - A_f} \quad (A_p < T < A_f), \tag{6b}$$

Integrating eq. (6) with the boundary conditions of $\xi(A_s) = 1$ and $\xi(A_f) = 0$ and the continuity condition that $\xi(A_p)$ should have same value from eqs. (6a) and (6b), we have

$$\xi = a_1(T - A_s)^2 + 1 \quad (A_s < T < A_p), \tag{7a}$$

$$\xi = a_2(T - A_f)^2 (A_p < T < A_f), \tag{7b}$$

where

$$a_1 = \frac{-1}{(A_f - A_s)(A_p - A_s)}, \quad a_2 = \frac{-1}{(A_f - A_s)(A_p - A_f)}.$$

Similarly during the phase transformation from austenite to twinned martensite, we have

$$\xi = m_1(T - M_s)^2 \quad (M_p < T < M_s). \tag{7c}$$

$$\xi = m_2(T - M_f)^2 + 1 \quad (M_f < T < M_p), \tag{7d}$$

where

$$m_1 = \frac{-1}{(M_s - M_f)(M_p - M_s)},$$

$$m_2 = \frac{-1}{(M_s - M_f)(M_p - M_f)}.$$

Eq. (7) is the phase transformation equation of SMA in the state of stress free. The equation can express the influences of phase transformation peak temperatures as well as phase transformation starting and finishing temperatures on the phase transformation behaviors of SMA.

Figure 3 shows the relationship of phase transformation critical stresses and temperature supposed by Brinson[6], where C_A, C_M, σ_s^{cr} and σ_f^{cr} are the material constants expressing the phase transformation behaviors of SMA. In order to establish the shape memory evolution equation which can express the influences of phase transformation peak temperatures on the phase transformation behaviors of SMA, we expand the relationship of phase transformation critical stresses and temperature from Figure 3 to Figure 4, where

$$\sigma_p^{cr} = (\sigma_f^{cr} - \sigma_s^{cr})\frac{M_p - M_s}{M_f - M_s} + \sigma_s^{cr}.$$

According to the phase transformation equation during the transformation from austenite to detwinned martensite, eqs. (7c) and (7d), and the relationship of phase transformation critical stresses and temperature, Figure 4, we can have the relationship of shape memory factor η, temperature T and equivalent stress $\bar{\sigma}$ during the transformation from austenite or twinned martensite to detwinned martensite:

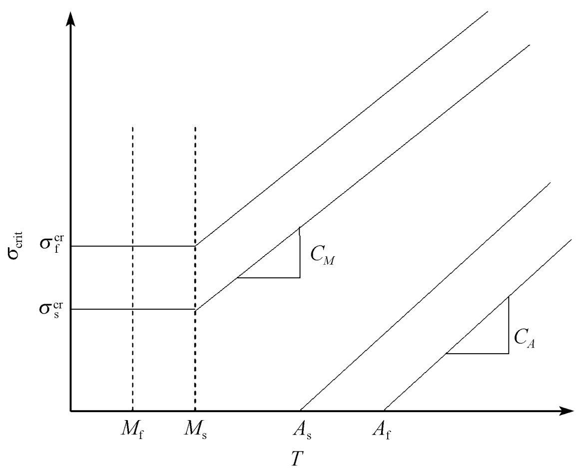

Figure 3 The Brinson's relationship of phase transformation critical stresses and temperature[6].

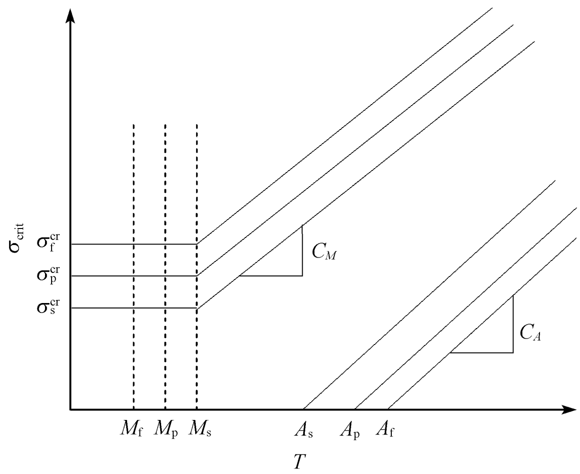

Figure 4 The new relationship of phase transformation critical stresses and temperature.

when $\sigma_s^{cr} + C_M(T - M_s)_+^1 < \bar{\sigma} < \sigma_p^{cr} + C_M(T - M_s)_+^1$,

$$\eta = s_1[\bar{\sigma} - \sigma_s^{cr} - C_M(T - M_s)_+^1]^2, \tag{8a}$$

when $\sigma_p^{cr} + C_M(T - M_s)_+^1 < \bar{\sigma} < \sigma_f^{cr} + C_M(T - M_s)_+^1$,

$$\eta = s_2[\bar{\sigma} - \sigma_f^{cr} - C_M(T - M_s)_+^1]^2 + 1, \tag{8b}$$

where the equivalent stress $\bar{\sigma} = \left[\sigma_{ij}^{/}\sigma_{ij}^{/}/2\right]^{1/2}$ and $\sigma_{ij}^{/}$ is deviatoric stress tensor. It is easy to prove that $\bar{\sigma} = \sigma$ in one-dimensional state.

$$s_1 = \frac{-1}{(\sigma_s^{cr} - \sigma_f^{cr})(\sigma_p^{cr} - \sigma_s^{cr})},$$

$$s_2 = \frac{-1}{(\sigma_s^{cr} - \sigma_f^{cr})(\sigma_p^{cr} - \sigma_f^{cr})},$$

$$(T - M_s)_+^1 = \begin{cases} T - M_s, & T > M_s, \\ 0, & T \leqslant M_s. \end{cases}$$

According to the phase transformation equation during the transformation from twinned martensite to austenite, eqs. (7a) and (7b), and the relationship of phase transformation critical stresses and temperature, Figure 4, we can have the relationship of shape memory factor η, temperature T and equivalent stress $\bar{\sigma}$ during the transformation from detwinned martensite to austenite:

when $T > A_s$ and $C_A(T - A_s) > \bar{\sigma} > C_A(T - A_p)$,

$$\eta = \eta_u \left\{ \frac{a_1}{C_A^2} [\bar{\sigma} - C_A(T - A_s)]^2 + 1 \right\}, \tag{8c}$$

when $T > A_s$ and $C_A(T - A_p) > \bar{\sigma} > C_A(T - A_f)$,

$$\eta = \eta_u \frac{a_2}{C_A^2} [\bar{\sigma} - C_A(T - A_f)]^2, \tag{8d}$$

where η_u is the initial value of shape memory factor during the process of unloading.

Eq. (8) is the new shape memory evolution equation of SMA. It can predict the phase transformations among austenite, twinned martensite and detwinned martensite effectively. Especially it overcomes the limitations that the previous phase transformation equations or shape memory evolution equations fail to express the transformation from twinned martensite to detwinned martensite and the influences of phase transformation peak temperatures on the phase transformation behaviors of SMA. So the new shape memory evolution equation can describe the phase transformation behaviors of SMA more precisely and comprehensively than the previous phase transformation equations or shape memory evolution equations.

3 Macro-mechanical constitutive equation

There are two available approaches to establish the constitutive relationship of stress, strain and temperature of SMA according to the theory of thermo-mechanics. One is to formulate the stress as the function of temperature and strain by using the expression of Helmholtz free energy. The other is to formulate the strain as the function of temperature and stress by using the expression of Gibbs free energy. In one-dimensional state, Gibbs free energy G is expressed as the function of stress σ, temperature T and shape memory factor η:

$$G = G(\sigma, \eta, T). \tag{9}$$

The relationship of strain ε and Gibbs free energy G is

$$\varepsilon = -\frac{\partial G(\sigma, \eta, T)}{\partial \sigma}. \tag{10}$$

Differential operation on eq. (10) leads to

$$\mathrm{d}\varepsilon = C\mathrm{d}\sigma + S\mathrm{d}\eta + \alpha \mathrm{d}T, \tag{11}$$

where

$$C = -\frac{\partial^2 G}{\partial \sigma^2},\ S = -\frac{\partial^2 G}{\partial \sigma \partial \eta},\ \alpha = -\frac{\partial^2 G}{\partial \sigma \partial T}.$$

are the material parameters related to elastic strain, shape memory strain and thermo-expanding strain respectively. Eq. (11) also reads as

$$\mathrm{d}\varepsilon = \mathrm{d}\varepsilon_e + \mathrm{d}\varepsilon_s + \mathrm{d}\varepsilon_t, \tag{12}$$

where

$$\mathrm{d}\varepsilon_e = C\mathrm{d}\sigma,\ \mathrm{d}\varepsilon_s = S\mathrm{d}\eta,\ \mathrm{d}\varepsilon_t = \alpha \mathrm{d}T,$$

ε_e, ε_s and ε_t are elastic strain, shape memory strain and thermo-expanding strain respectively.

According to eq. (12) the strain of SMA includes three parts of elastic strain, shape memory strain and thermo-expanding strain, which is expressed as

$$\varepsilon = \varepsilon_e + \varepsilon_s + \varepsilon_\alpha. \tag{13}$$

From Hooke's law, the elastic strain:

$$\varepsilon_e = \frac{\sigma}{E}, \tag{14}$$

where E is the elastic modulus of SMA. It is the function of martensitic volume fraction ξ, expressed as[6]

$$E = E_a + (E_m - E_a)\xi, \tag{15}$$

where E_a and E_m are the elastic moduli of SMA in the states of full austenite and full martensite, respectively.

Based on the foregoing definition of shape memory factor and its physical meaning, the shape memory strain of SMA:

$$\varepsilon_s = \varepsilon_L \eta, \tag{16}$$

where ε_L is the maximum residual strain, a material constant of SMA. It can be determined through tensile test at the temperature below austenite starting temperature[14]. In this paper ε_L is called as the maximum shape memory strain, which is more clear and direct than the maximum residual strain.

The thermo-expanding strain of SMA is expressed as

$$\varepsilon_t = \alpha(T - T_0), \tag{17}$$

where α is the thermo-expanding coefficient, and T_0 is the initial value of temperature T.

Substituting eqs. (14)—(17) into eq. (13), we have

$$\varepsilon = \frac{\sigma}{E_a + (E_m - E_a)\xi} + \varepsilon_L \eta + \alpha(T - T_0). \tag{18}$$

This is the macro-mechanical constitutive equation of SMA in one-dimensional state. In this paper, SMA is assumed as isotropic material. So eq. (18) can be expanded from one-dimension to three-dimension, expressed as

$$\varepsilon_{ij} = \frac{(1+v)\sigma_{ij}}{E_a + (E_m - E_a)\xi} - \frac{v\sigma_{kk}\delta_{ij}}{E_a + (E_m - E_a)\xi}$$

$$+\varepsilon_{ij}^{L}\eta+\alpha(T-T_0)\delta_{ij}, \tag{19}$$

where v, δ_{ij} and ε_{ij}^{L} are Poisson's ratio, Kronecker delta and the extreme shape memory strain tensor, respectively. The extreme shape memory strain tensor is expressed as

$$\varepsilon_{ij}^{\mathrm{L}}=T_{\alpha i}T_{\alpha j}\varepsilon_{\mathrm{L}}, \tag{20}$$

where $T_{\alpha i}/T_{\alpha j}$ is coordinate transformation tensor, α stands for the principle coordinate, and i/j stands for the arbitrary coordinate.

The new constitutive equation, eq. (19), and the new shape memory evolution equation, eq. (8), compose the new three-dimensional macro-mechanical constitutive model of SMA. It can predict the phase transformation behaviors and the thermo-mechanical processes realizing the shape memory effect of SMA more precisely and comprehensively. This is because it both expresses the influences of phase transformation peak temperatures on the phase transformation behaviors and the transformation from twinned martensite to detwinned martensite of SMA. In addition the new constitutive model is established on the basis of definition of shape memory factor and its physical meaning related to the microscopic mechanism of phase transformations of SMA, which unifies the macroscopic description and the microscopic mechanism of SMA. All material constants in the new constitutive can be determined through macroscopic experiments, which make it more easily used in practical applications than the previous three-dimensional constitutive models of SMA. For example the new constitutive model is easily used to develop the equations of finite element or to establish the library function of material called by the finite element software of ABAQUS or ANSYS, which can realize the numerical simulation analysis on SMA in the state of complex stress.

4 Numerical examples

The new constitutive model, which includes the shape memory evolution equation, eq. (8), and the mechanical constitutive equation, eq. (19), is used to numerically predict the phase transformation behaviors and the thermo-mechanical processes realizing shape memory effect of SMA. The numerical results are compared to those predicted by Brinson's constitutive model. The material constants of SMA used in the numerical calculations are listed in Table 1.

Table 1 Material constants used in numerical calculations[6,14]

A_s (℃)	A_p (℃)	A_f (℃)	E_a (GPa)	C_A (MPa/℃)
63.28	79.53	82.91	230.00	13.80
M_s (℃)	M_p (℃)	M_f (℃)	E_m (GPa)	C_M (MPa/℃)
49.47	43.81	36.21	67.00	8.00
ε_L (%)	σ_s^{cr} (MPa)	σ_f^{cr} (MPa)		
6.70	100.00	170.00		

Figure 5 shows the martensitic volume fraction-temperature curves of SMA in the state of stress free described by the new shape memory evolution equation, eq. (8), coupled with the relationship of martensitic volume fraction and shape memory factor, eq. (1), and Brinson's phase transformation equation respectively. In the state of stress free the phase transformation of SMA is between austenite and twinned martensite. During the process of heating SMA starts to transform from twinned martensite to austenite when the temperature is above austenite starting temperature and becomes full austenite when the temperature is above austenite finishing temperature. During the process of cooling SMA starts to transform from austenite to twinned martensite when the temperature is below martensite starting temperature and becomes full twinned martensite when the temperature is below martensite finishing temperature. It is well known that the curve shapes during the phase transformation from austenite to twinned martensite and from twinned martensite to austenite are the same according to Brinson's phase transformation equation but they are different according to the new shape memory evolution equation. However the results of DSC test, shown in Figure 2, reveal that the shapes of heat flow-temperature curve during the phase transformations from twinned martensite to austenite and from austenite

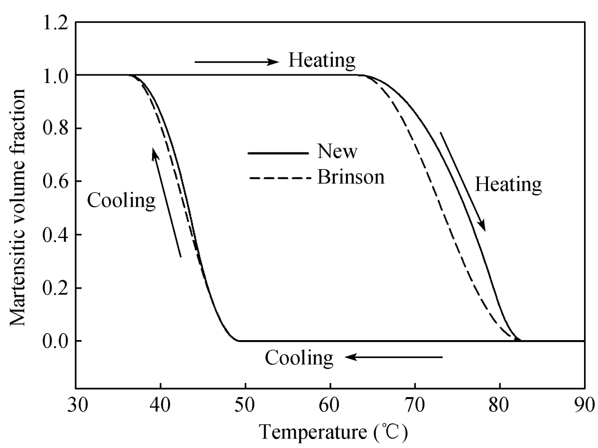

Figure 5 Martensitic volume fraction-temperature curves.

to twinned martensite are different. One is unsymmetric and the other is symmetric, which implies the shape of martensitic volume fraction/shape memory factor-stress during the phase transformation from austenite to twinned martensite and from twinned martensite to austenite should be different. So the new shape memory evolution equation predicts the phase transformation behaviors of SMA more precisely than Brinson's phase transformation equation. This is because the new shape memory evolution equation can express the influences of phase transformation peak temperatures as well as phase transformation starting and finishing temperatures, while Brinson's phase transformation equation only expresses the influences of phase transformation starting and finishing temperatures.

Figure 6 shows the martensitic volume fraction/shape memory factor-stress curves respectively described by the new shape memory evolution equation, eq. (8), coupled with the relationship of martensitic volume fraction and shape memory factor, eq. (1), and Brinson's phase transformation equation at the temperature of 90℃ which is above austenite finishing temperature. The initial state of SMA is full austenite. During the process of loading SMA starts to transform from austenite to detwinned martensite when the stress is above martensite starting critical stress, and becomes full detwinned martensite when the stress is above martensite finishing critical stress. Both shape memory factor and martensitic volume fraction increase from 0 to 1 with the transformation from austenite to detwinned martensite. Detwinned martensite is unstable at the temperature above austenite finishing temperature. During the process of unloading SMA starts to transform from detwinned martensite to austenite when the stress is below austenite starting critical stress, and becomes full austenite when the stress is below austenite finishing critical stress. Both shape memory factor and martensitic volume fraction decreases from 1 to 0 with the transformation from detwinned martensite to austenite. Similarly to Figure 5, the main difference between the descriptions of new shape memory evolution equation and Brinson's phase transformation equation is in the range of the transformation to austenite. This also proves the new shape memory evolution equation describes the phase transformation more precisely than Brinson's phase transformation equation according to the foregoing discussions on Figure 5.

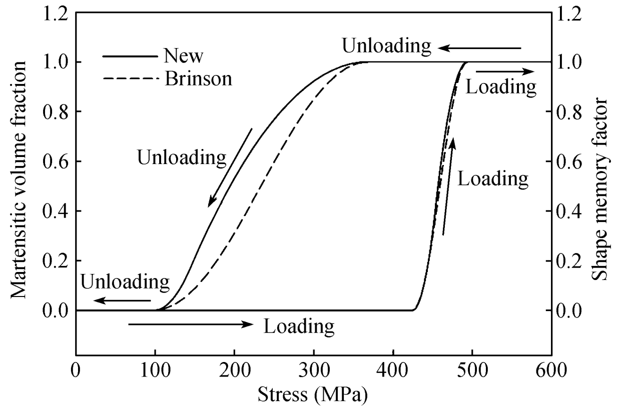

Figure 6 Martensitic volume fraction/shape memory factor-stress curves.

Figure 7 shows the martensitic volume fraction/shape memory factor-stress curves of SMA described by the new shape memory evolution equation, eq. (8), coupled with the relationship of martensitic volume fraction and shape memory factor, eq. (1), at the temperature of 30℃ which is below martensite finishing temperature. The initial state of SMA is full twinned martensite. The initial values of shape memory factor and martensitic volume fraction are 0 and 1 respectively. During the process of loading SMA starts to transform from twinned martensite to detwinned martensite when the stress is above martensite starting critical stress, and becomes full detwinned martensite when the stress is above martensite finishing critical stress. Shape memory factor increases from 0 to 1 with the transformation from twinned martensite to detwinned martensite. Martensitic volume fraction does not change, keeping the value of 1. During the process of unloading there is no phase transformation occurring in SMA. Both shape memory factor

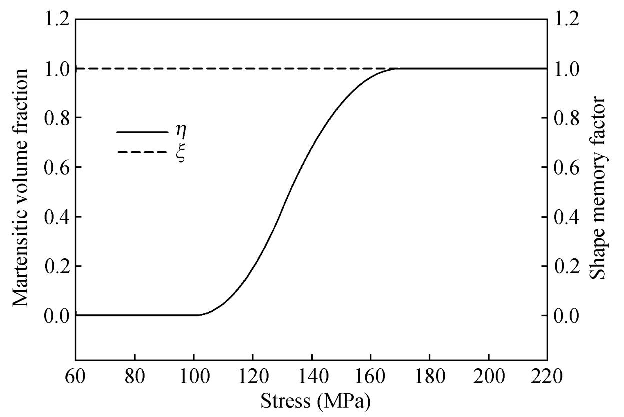

Figure 7 Martensitic volume fraction/shape memory factor-stress curves.

and martensitic volume fraction do not change, keeping the value of 1, which is due to that the detwinned martensite is stable at the temperature below austenite starting temperature.

Figure 8 shows the stress-strain curves of SMA at different temperatures described by the new constitutive model, eqs. (8) and (19), and Brinson's constitutive model respectively. The initial states of SMA are full austenite at the temperatures of 95, 75 and 55℃ and full twinned martensite at the temperature of 35℃. During the process of loading SMA initiates the linear elastic deformation with the elastic modulus of full austenite (at the temperatures of 95, 75 and 55℃) or full martensite (at the temperature of 35℃). Shape memory strain of SMA starts to appear at martensite starting critical stress, and rises to its maximum value at martensite finishing critical stress. The phase transformation critical stresses of SMA increase with the increasing of temperature. During the process of unloading SMA initiates the linear elastic deformation with the elastic modulus of full martensite. At the temperature of 95℃, which is above austenite finishing temperature, shape memory strain starts to recover at austenite starting critical stress and recover completely at austenite finishing critical stress. After unloading there is no residual strain in SMA, which expresses as the shape memory effect induced by unloading. At the temperature of 75℃, shape memory strain starts to recover at austenite starting critical stress. After unloading SMA becomes the mixed state of austenite and detwinned martensite. Shape memory strain partly remains as the residual strain, which can be recovered through heating to austenite finishing temperature. This is the shape memory effect induced by unloading and temperature. At the temperatures of 55 and 35℃, which are below austenite starting temperature, the shape memory strain does not recover because detwinned martensite of SMA is stable at the temperature below austenite starting temperature. The shape memory strain becomes the residual strain after unloading. It can be recovered through heating to austenite finishing temperature, which expresses the shape memory effect induced by temperature. We can see that the main difference between the descriptions of the new and Brinson's constitutive models is in the process of shape memory strain recovering, i.e. the process of the transformation to austenite, during the unloading. This can prove the new constitutive model predicts the thermo-mechanical processes of SMA more precisely than Brinson's constitutive model according the foregoing discussions on Figures 6 and 5.

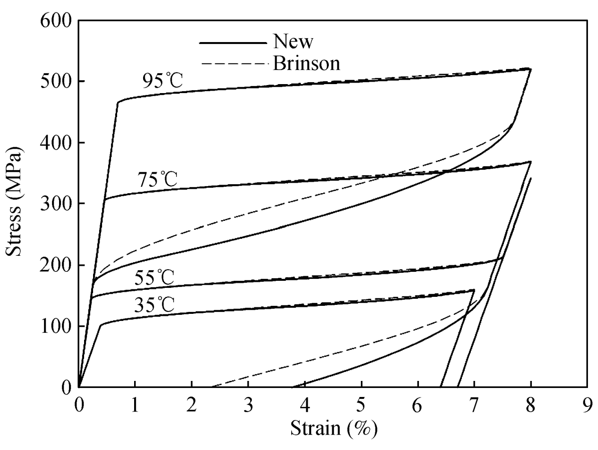

Figure 8 Stress-strain curves at different temperatures.

5 Conclusions

The mechanisms of super elasticity and shape memory effect of SMA are explained with the comparison between macroscopic phenomena and microscopic structures by using the shape memory factor defined in ref. [12]. The conventional super elasticity and shape memory effect of are further unified as shape memory effect. The concept of shape memory factor is redefined to make clear its physical meaning. The new shape memory evolution equation is developed on the basis of the differential relationship of martensitic volume fraction and phase transformation free energy and the results of DSC test. It overcomes the limitations that the previous phase transformation equations or shape memory evolution equations fail to express the transformation from twinned martensite to detwinned martensite and the influences of phase transformation peak temperatures on the phase transformation behaviors of SMA. The new macro-mechanical constitutive equation, which predicts the thermo-mechanical processes realizing the shape memory effect of SMA, is established based on the expression of Gibbs free energy. It is expanded from one-dimension to three-dimension with assuming SMA as isotropic material. All material constants in the new constitutive equation can be determined through macroscopic experiments, which makes it more easily used in practical applications than the previous three- dimensional constitutive equation of SMA. Numerical results reveal the new constitutive model, which includes the new shape memory evolution equation and the new macro-mechanical constitutive equation, predicts the phase transformation behaviors and the thermo-mechanical processes realizing shape memory effect more precisely and comprehensively than the previous constitutive models of SMA.

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1 Yang D Z. Intelligent Materials and Systems (in Chinese). Tianjin: Tianjing University Press, 2000. 104－142

2 Tanaka K. A thermomechanical sketch of shape memory effects: One-dimension tensile behavior. Res Mech, 1986, 18: 251－263

3 Liang C, Rogers C A. One-dimensional thermomechanical constitutive relations for shape memory materials. J Intell Mater Syst Struct, 1990, 1: 207－234[DOI]

4 Boyd J G, Lagoudas D C. Thermomechanical response of shape memory composites. J Intell Mater Syst Struct, 1994, 5: 333－346[DOI]

5 Sun Q P, Hwang K C. Micromechanics modeling for the constitutive behaviors of polycrystalline shape memory alloys. J Mech Phys Solids, 1993, 41: 1－33[DOI]

6 Brinson L C. One-dimension constitutive behavior of shape memory alloys: Thermomechanical derivation with non-constant material functions and martensite internal variable. J Intell Mater Syst Struct, 1993, 4: 229－242[DOI]

7 Peng X, Yang Y, Huang S. A comprehensive description for shape memory alloys with a two-phase constitutive model. Int J Solids Struct, 2001, 38: 6925－6940[DOI]

8 Zhu Y G, Lv H X, Yang D Z. Constitutive model of shape memory alloys (in Chinese). Chin J Mater Res, 2001, 15(6): 263－268

9 Brocca M, Brinson L C, Bazant Z P. Three-dimension constitutive model for shape memory alloys based on microplane model. J Mech Phys Solids, 2002, 50: 1051－1077[DOI]

10 Li H, Peng X, Huang S. A conventional plasticity based two-phase constitutive model for shape memory alloys (in Chinese). Acta Mech Solida Sin, 2004, 25(1): 58－62

11 Guo Y, Liu F, Dai X, et al. Dynamic pseudoelastic behavior of TiNi alloys and a strain rate dependent phase transition constitutive model (in Chinese). Explos Shock Waves, 2003, 23(2): 105－110

12 Zhou B, Wang Z, Liang W. A micromechanical constitutive model of shape memory alloys (in Chinese). Acta Metall Sin, 2006, 42(9): 919－924

13 Zhou B, Yoon S H. A new phase transformation constitutive model of shape memory alloys. Smart Mater Struct, 2006, 15: 1967－1973[DOI]

14 Zhou B, Yoon S H. Study on phase transformation and thermomechanical behaviors of Ni-Ti shape memory alloy. In: Batra R C, Qian L F, Zhang Y L, et al., eds. Proceeding of International Conference on Mechanical Engineering and Mechanics. Beijing: Science Press, 2005. 1001－1005

15 Zhou B, Wan Z Q, Yoon S H, et al. Mechanical behaviors of torsion actuator of shape memory alloy. Key Eng Mater, 2008, 385-387: 213－216[DOI]

16 Xiong K, Shen W. Experimental research on the relationship between SMA wire wound angle and SMA torsion actuator primary performances (in Chinese). Chin J Mech Eng, 2003, 39(12): 123－128