AIP | Journal of Applied Physics

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Citation: J. Appl. Phys. **112**, 033519 (2012); doi: 10.1063/1.4745023
View online: http://dx.doi.org/10.1063/1.4745023
View Table of Contents: http://jap.aip.org/resource/1/JAPIAU/v112/i3
Published by the American Institute of Physics.

Voltage-induced deformation in dielectric

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(Received 7 May 2012; accepted 11 July 2012; published online 9 August 2012)

When subjected to a high electric field, dielectrics deform. For a hard dielectric, voltage-induced deformation is usually small, and the potential for deformation is determined by its breakdown voltage. If a dielectric is soft, the voltage-induced deformation is significantly large. The potential for deformation is determined by the breakdown voltage together with electromechanical instability and snap-through instability. Based on the theoretical researches conducted by Zhao and Suo [Phys. Rev. Lett. **104**, 178302 (2010)] and Li *et al.* [Int. J. Smart Nano Mater. **2**(2), 59-67 (2011)], taking the dielectric elastomer soft material as research object, we introduce three kinds of material limits: strain-stiffening, polarization saturation, and breakdown voltage. The effect of material limits on voltage-induced deformation is analyzed. Also investigated is the influence of pre-stretch and material parameters on the deformation. For a specific soft dielectric material under certain pre-stretch, the theoretical maximum electrical actuation deformation can be determined. [http://dx.doi.org/10.1063/1.4745023]

I. INTRODUCTION

For a long time, researchers have been devoting themselves to the study of widely used hard materials in engineering fields. However, the prevalent mechanical structures of organisms including all animals and plants in the natural world are usually composed of soft materials.[1,2] Compared with traditional hard materials represented by metals and ceramics, polymer is a typical representative of soft materials. Soft materials can produce deformation at varying degrees to show its active characteristic when subjected to stimuli outside, such as mechanical force, electrical field, and temperature.

Subjected to high electric field, dielectrics can deform. If a dielectric is hard (such as ceramic), the voltage-induced deformation is trivial and determined by the breakdown voltage. If a dielectric is soft (such as rubber), the voltage-induced deformation is large and determined by the breakdown voltage together with electromechanical instability and snap-through instability.[25]

Intelligent soft material is a kind of new functional materials which can respond to external stimuli; after appropriate judgment and treatment, it is executable itself. Compared with traditional hard smart materials, these cheap and light materials are capable of large deformation and excellent biological compatibility. Intelligent materials and intelligent devices have huge potential applications from the exploit and encapsulation of oil to the delivery of medicines. They are widely used in mechanical fields, medical treatment, and war industry. Because the deformation ability and driving force of the materials are close to those of biological muscles, they also have rather bright prospect in the robot field.

Electroactive polymer (EAP) is a kind of typical soft active dielectric materials. When subjected to an electric field, it constricts in thickness and expands in area, but once the electric field is revoked, it will return to the original shape. This process is known as electrical actuation. Based on this characteristic, EAP can be used in intelligent transducers including actuators, sensors, and energy harvest devices.[1–4] According to different working mechanisms, EAP can be divided into two categories: electronic-EAP and ionic-EAP. Electronic-EAP requires high inducing electric field to achieve effective electrical actuation deformation. Ionic-EAP is made up of electrodes as well as electrolytes and works in a humid environment with its surface kept moist.

Dielectric elastomers (DEs) belong to the electronic EAP, which can also be called artificial muscle of electroactive polymer. DEs compose a category of deformable soft dielectrics. Based on its outstanding merits, including large deformation ability, capacity for high elastic energy density and conversion efficiency, short response time, light weight, and low price, DEs can be used to manufacture actuators, sensors, energy harvesters, Braille displays, and adaptive optical elements which can be widely used in the fields of artificial muscles, smart robots, biomedicine, micro-mechanical electronics, aerospace, and so on.[2–4]

Zhenyi *et al.* observed an actuation strain of about 3% of thermoplastic polyurethane in 1994, under an electric field of 20 MV/m.[7] In 1998, Pelrine *et al.* found that the silicone rubber could exhibit an actuation strain of about 30% when subjected to an electric field.[8] In 2000, Science reported that after 300% equal-biaxial pre-stretch the acrylic acid can produce 100% actuation strain under certain electric field.[1] Recent years, actuators and sensors with several structures of EAP

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dielectric elastomers are designed and manufactured.[1–8] However, deformation scales of these actuators still can not meet the needs of practical applications. The actuation strains of the roll actuators, stacked actuators, and folded dielectric elastomer actuators can hardly exceed 10% in general.

One of the constraints in the applications mentioned above lies in the inadequate actuation deformation of dielectric elastomers. Although the deformation is large to a certain degree (100%), it does not fully meet the needs of commercial applications. Consequently, theoretical prediction of the super actuation deformation of the dielectrics is quite necessary for us to guide further experimental research.

The large deformation, constitutive relation, electromechanical instability, snap-through instability, failure analysis, description of stability area, analysis of inhomogeneous large deformation, dynamics analysis, as well as giant electrical actuation deformation mechanisms of dielectric elastomers are primary emphases of theoretical investigation concerning the dielectrics.[9–55]

As to an ideal dielectric elastomer, Zhao and Suo analyzed its electromechanical stability using the neo-Hookean elastic strain energy.[10] Díaz-Calleja researched stable and unstable areas of a neo-Hookean-type ideal silicon rubber dielectric elastomer.[15] Xu *et al.* investigated electromechanical stability of dielectric elastomers within the resultant stress theory.[46]

As to linear and nonlinear dielectric elastomers, Zhao *et al.* studied the mechanical properties and electromechanical stability behavior of electromechanical coupling system in dielectric elastomers undergoing large deformation, with influence of large electrostriction deformation on the permittivity considered.[29] Liu *et al.* deduced the analytical expressions of critical parameters based on critical controling equations of dielectric elastomers' electromechanical stability.[19] He *et al.* built the state equations of ideal dielectric elastomer films undergoing inhomogeneous deformation.[30]

As to the investigation of giant deformation of dielectric elastomers, Suo *et al.* studied the mechanical behavior, large deformation behavior, and stability behavior of dielectric elastomers of interpenetrating networks.[26] Zhao and Suo built a dielectric elastomer theory with giant electrical actuation deformation and predicted that dielectric elastomers capable of giant electrical actuation deformation can be manufactured.[25] Furthermore, Koh *et al.* promoted the mechanism of dielectric elastomers' giant deformation.[55]

As to the investigation of dielectric elastomer converters, Zhu *et al.* investigated the finite deformation and electromechanical stability of a tubular DE driver.[44] Moscardo *et al.* put forward a typical failure model of dielectric elastomer scroll drivers.[27] Koh *et al.* analyzed the failure model of dielectric elastomer energy harvesters, calculated the electrical energy produced in a cycle and the maximum efficiency.[17]

Suo summarized the basic theory of dielectric elastomers in detail, developed the continuum mechanics and thermodynamics of dielectric elastomers with coupled large deformation and electrical field, and described the nonlinear and nonequilibrium state behaviors, such as electromechanical instability and viscoelasticity behavior. What is more! The finite element method is also applied to the numerical analysis of dielectric elastomer devices.[2]

This paper, which is based on the theoretical research conducted by Suo[25] and Li,[42] taking dielectric elastomers as an example, studied the influences of three material limits, including strain stiffening, polarization saturation, and breakdown voltage, as well as of a important material parameter, pre-stretch, on the dielectrics' actuation deformation. For a specific soft active dielectric material which is pre-stretched, its maximum actuation deformation can be determined theoretically.

II. MATERIAL LIMIT

A. Strain-stiffening

Rubber-like materials can exhibit effects of strain-stiffening.[25,26,42,47,56,57] In an elastomer, each individual polymer chain has a finite contour length. When the elastomer is not subject to any load, the polymer chains are coiled, allowing a large number of conformations. When stretched, the end-to-end distance of each polymer chain increases and eventually approaches the finite contour length, setting up a limiting stretch. On approaching the limiting stretch, the elastomer stiffens steeply. When subjected to a mechanical force and approaching the limiting stretch, λ_{lim}, the elastomer stiffens sharply.

B. Polarization saturation

Dielectrics can experience effects of polarization saturation.[42,47,57] Each elastic dielectric chain contains electric dipoles. When no external electric field is applied, the dipoles, affected by temperature, are along arbitrary directions and exhibit a chaotic arrangement. When a low voltage is applied to the elastomer, the original disordered dipoles will realign along the direction of the electric field. If the voltage is large enough, an arrangement composed of perfectly realigned dipoles can be yielded and the polarization of the elastomer is saturated.

C. Electric breakdown

After imposing a voltage on a dielectric, if the dielectric is stiff, even little deformation may lead to electrical breakdown (EB), while for a soft dielectric, EB can not be induced until the dielectric is deformed to a certain giant degree. The breakdown voltages of the two kinds of dielectrics are different in their determinations. Applying voltage to the dielectric, the corresponding voltage changes little with the deformation if the dielectric is hard, whereas for a soft material, the voltage changes a lot with the large deformation.

III. ELECTROMECHANICAL STABILITY AND SNAP-THROUGH STABILITY

A dielectric elastomer transducer is indeed a thin membrane of polymer, sandwiched between compliant electrodes. Dielectric elastomer soft materials usually undergo electromechanical instability. In the experiments conducted by Plante *et al.*, the instability phenomenon of deformable dielectric undergoing mechanical force and electrical force coupling field was observed.[58] Under certain voltage, the dielectric will deform, with part of the thin film smooth and the other parts wrinkled. Because the voltage of each point is the same, the two states exist at the same time. In the wrinkled region, the film suffers from larger deformation and become thinner. The experiment above suggests that when suffering from electromechanical coupling effect, the electrical breakdown of the deformable dielectric soft materials, which is induced by the electromechanical instability, will occur. The specific process is as follows. When subjected to a voltage, a dielectric elastomer will reduce in thickness and expands in area, leading to increase of the electric field applied to the membrane. This positive feedback continues until the electric field reaches the critical electric field and the dielectric elastomer breakdown. This process is called the electromechanical instability.[10,13–16] Typical voltage-strain relationship of a deformable dielectric undergoing the electromechanical stability is as follows.[10,14] As the increase of the strain, the voltage increases. When the strain approaches its limit, λ_C, the voltage approaches the breakdown voltage of the dielectric, and the electromechanical instability appears.

When subjected to a voltage, due to strain stiffening of the elastomer, the deformable dielectric may reach the local maximum at first when the stretch reaches λ_C, and electromechanical instability will not appear. As increase of the voltage, the deformable dielectric experiences snap-through,[55,57] and the dielectric remains stability until the deformation reaches λ_{lim} (the maximum deformation).

This voltage-stretch curve of the deformable dielectric reaches the local maximum first and then decreases; while approaching the limiting stretch λ_{lim}, the curve increases steeply. This process presents the snap-through stability of the deformable dielectric. Obviously, the voltage-stretch curve is not monotonic.

A. The influence of pre-stretch on electrical deformation

As we know, exertion of pre-stretch can improve electromechanical stability of the deformable dielectric significantly.[10,16] The snap-through stability of a pre-stretched deformable dielectric is as follows. The voltage-stretch response of the dielectric elastomer can be modified by applying appropriate pre-stretch λ_p. At this point, the curve increases monotonically, the local maximum of the curve disappears, and deformable dielectric reaches a steady state when the curve approaches the limiting stretch λ_{lim}. This indicates that a pre-stretched deformable dielectric can achieve larger actuation deformation when experiencing the snap-through stability.

B. The influence of polarization saturation on electrical deformation

Polarization saturation of the deformable dielectric, which we have introduced above, can also have obvious effect on the voltage-stretch curve. When the saturated electric displacement $D_s \to \infty$, the voltage-stretch curve in Figure 1(a) regardless of the influence of the polarization saturation. Such as Figure 1(a), the local critical stretch will be 1.26 if we use the neo-Hookean elastic energy model, and the local critical stretch will be 1.37 if we use the Mooney-Rivlin elastic energy model, which are quite fit with the experiment results.[7,10,14] However, when the saturated electric displacement is a suitable finite value, the local maximum of the voltage-stretch curve disappeared. The curve increases monotonically until reaching the limiting stretch λ_{lim} and being stabilized. In Figure 1(b), we predict that designing and adjusting suitable stress-strain curve of materials can lead to the snap-through instability of deformable dielectric and then to avoid the electrical breakdown. It is also been proved by the experiments of Christoph.[53]

C. The influence of pre-stretch and polarization saturation on electrical deformation

Figure 2 demonstrates the snap-through stability of the pre-stretched and polarization-saturated deformable dielectric. By applying an external force before actuation of the dielectric elastomer, the voltage-stretch curve is modified, and the dielectric can exhibit larger deformation. The local maximum of the voltage-stretch curve is disappearing when $D_s \to \infty$. However, when D_s is a finite value, by applying an external force before actuation of the dielectric elastomer we will recognize that the slope of the first half of the original

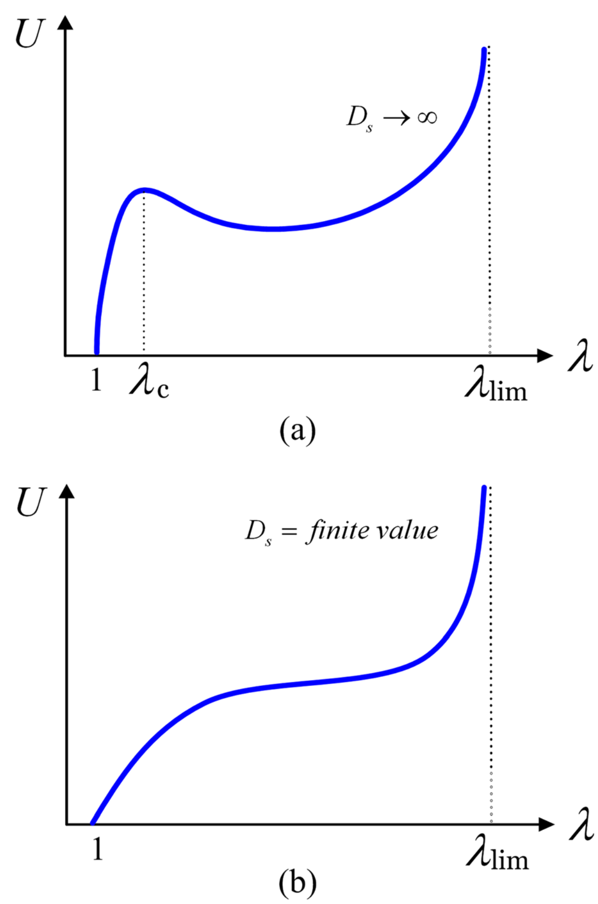

FIG. 1. The snap-through stability of deformable dielectric undergoing polarization saturation.

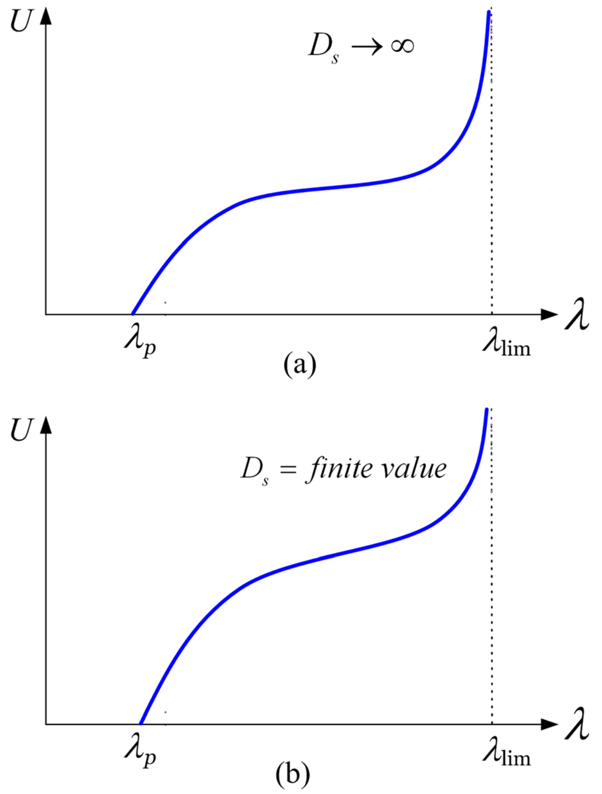

FIG. 2. The snap-through stability of deformable dielectric undergoing pre-stretch and polarization saturation.

curve is larger than that of the second half and the voltage at the inflection point is higher.

D. The influence of chain length on electrical deformation

The voltage-deformation relationships of two typical deformable dielectric polymers with short chains and long chains respectively are as follow.[55] The polymer with short chains has lower limiting stretch λ_{lim} which has great effect on the voltage-deformation curve. If λ_{lim} is low enough, the local maximum of the voltage-deformation curve disappears. Polymers with short chains include silicone rubber, dielectric elastomers, dielectric elastomers with interpenetrating networks, and so on. However, shows that polymers with long chains tend to possess higher limiting stretch λ_{lim} and that there will exist a local maximum in the voltage-stretch curve. These effects are similar to those of polarization saturation we have previously given when $D_s \to \infty$. Acrylic dielectric elastomer is a typical polymer with long chains.

IV. DETERMINATION OF ELECTRICAL ACTUATION DEFORMATION

When no pre-stretch (meaning absence of mechanical force) or little pre-stretch (which has slight effect on electrical actuation curve) is applied to a dielectric, the typical voltage-stretch curve firstly increases before reaching the local maximum and then decreases until eventually jumping up suddenly. Breakdown voltage decreases monotonically with increase of stretch.[55]

If the electrical deformation curve and electrical breakdown curve of a deformable dielectric intersect before the local apex of the former curve, the dielectric will have been broken down before it reaches the electromechanical instability which is usually undergone by hard dielectrics. At this time the dielectric produces small electrical actuation deformation of λ_b.

If the two curves of the deformable dielectric intersect after the local maximum of the former curve, the deformable dielectric will fail because of electromechanical instability. The dielectric does not experience the snap-through deformation. At this time the dielectric exhibits large electrical actuation deformation of λ_c.

If the deformable dielectric does not fail at the local maximum which represents the critical point of electromechanical instability (for effect of strain stiffening), the deformable dielectric experiences the snap-through deformation. Accordingly, the dielectric produces large electrical actuation deformation of λ_b.

Figure 3 reveals the electromechanical response curve of another representative deformable dielectric. Through applying large pre-stretch or taking polarization saturation into consideration, the electrical actuation curve increases monotonically. The deformable dielectric does not experience the electromechanical

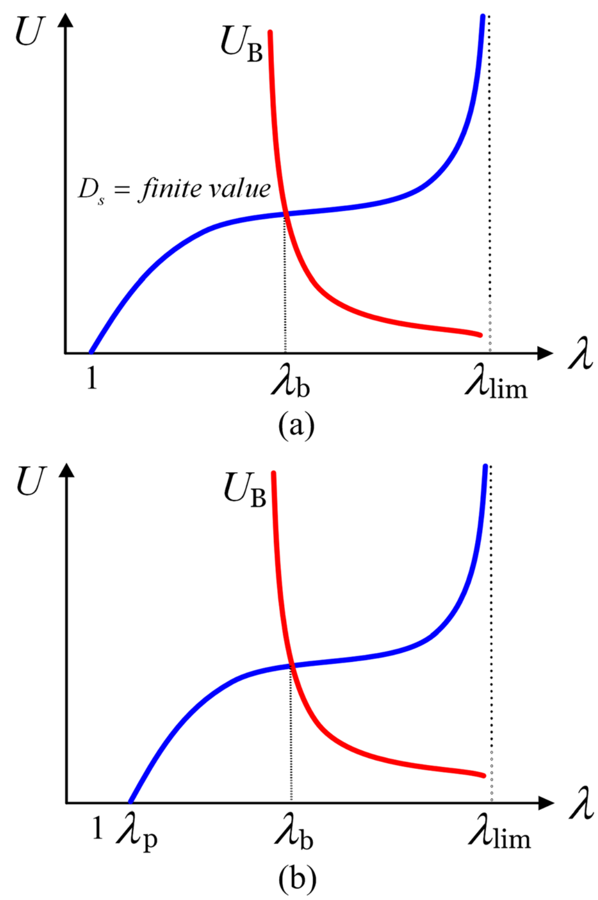

FIG. 3. The electromechanical response of deformable dielectric undergoing polarization saturation (D_s is a limited value) or with large pre-stretch. At this time, the electrical actuation deformation curve increase monotonously, in the voltage-deformation plane, the maximum electrical actuation deformation is determined by the relative position of breakdown voltage and snap-through stability.

instability. The electrical actuation is determined by breakdown voltage. At the same time, the dielectric produces large deformation of λ_b. The previous experiments introduced above have explained this type of deformation wonderfully. The experimental facts include that pre-stretched acrylic dielectric elastomer can reach actuation strain of 100% and that interpenetrating networks dielectric elastomers can also achieve super electrical actuation deformation. On the other hand, Figure 3 represents the electromechanical response of a deformable dielectric with short chains.

V. EXAMPLES

We now take the Gent elastic strain energy model as an example to investigate the deformation and stability behavior of dielectric elastomers undergoing polarization saturation.

A. Gent elastic strain energy model

In an elastomer, each individual polymer chain has a finite contour length. When the elastomer is not subjected to any loads, the polymer chains are coiled, allowing a large number of conformations. When subjected to loads, the polymer chains become less coiled. As the loads increase, the end-to-end distance of each polymer chain approaches the finite contour length, and the elastomer approaches a limiting stretch. While approaching the limiting stretch, the elastomer stiffens steeply. This effect is absent in the Mooney-Rivlin or neo-Hookean model but is represented by the Arruda and Boyce model[26] and by the Gent model with three-parameters.[56] The Gent model can be expressed in the form of

$$U(\lambda_1,\lambda_2)=\frac{\mu}{2}[-\alpha J_{lim}\log\left(1-\frac{J_1}{J_{lim}}\right)+(1-\alpha)J_2], \quad (1)$$

where J_1, equal to $\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3$, is a scalar measure of strain, J_2, equal to $\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3$ and independent of J_1, is a second symmetric measure of strain, $J_{\lim}$ is a constant related to the limiting stretch and equals $\lambda_{1\lim}^2+\lambda_{2\lim}^2+\lambda_{1\lim}^{-2}\lambda_{2\lim}^{-2}-3$ (for rubber, typical values of the dimensionless parameter $J_{\lim}$ for simple extension range from 30 to 100 (Ref. 56)), μ is the small-strain shear modulus for infinitesimal deformations, and α is a dimensionless constant satisfying $0<\alpha\le 1$.

When $\alpha=1$, Equation (1) becomes that $U(\lambda_1,\lambda_2)=-\frac{\mu}{2}J_{lim}\log(1-\frac{J_1}{J_{lim}})$, and the Gent model with three parameters can be simplified to a two-parameter model. When $J_1/J_{\lim}\to 0$, the Taylor expansion of Eq. (1) gives that $U=\frac{\mu}{2}J_1$. That is, the Gent model with two parameters regresses to the neo-Hookean model when deformation is small compared to the limiting stretch. A molecular basis for the basic Gent model was given. It is shown that the infinitesimal shear modulus can be expressed as $\mu=nkT$, as is usual in the molecular models, where k is the Boltzmann constant, T is the absolute temperature, and n is the chain density, while $J_m=3(N-1)$, where N is the number of links in a single chain.

When $J_1/J_{\lim}\to 1$, the elastomer approaches the limiting stretch. We can further expand the natural logarithm of the Gent model as $U(\lambda_1,\lambda_2)=\frac{\mu}{2}[(I_1-3)+\frac{1}{2J_{\lim}}(I_1-3)^2+\ldots+\frac{1}{(n+1)J_{\lim}{}^n}(I_1-3)^{n+1}]$. A general form is $U(\lambda_1,\lambda_2)=\sum_{i=1}^{n}C_i(J_{\lim})(I_1-3)^i$, which can be regarded as a expression of the Rivlin elastic strain energy model, where $U(\lambda_1,\lambda_2)=\sum_{i,j=0}^{\infty}C_{ij}(I_1-3)^i(I_2-3)^j$ with $j=0$.

When $\alpha\ne 1$, $J_1/J_{\lim}\to 0$, the Taylor expansion of Eq. (1) gives that $U=\frac{\mu}{2}[\alpha J_1+(1-\alpha)J_2]$. In fact, the Gent model with three-parameters degenerate into the Mooney-Rivlin model when deformation is small compared to the limiting stretch.

B. Special electric energy of dielectric elastomers

When the model of an ideal dielectric elastomer was proposed, the elastomer was considered as a linear dielectric with $E=D/\varepsilon$, where ε is the permittivity. To study the effect of polarization saturation, we, in the paper, assume that the elastomer is a nonlinear dielectric which is characterized by the function below[42]

$$D=D_s\tanh(\varepsilon E/D_s), \quad (2)$$

where D_s is the saturated electric displacement. When electric field is low, $\varepsilon E/D_s\ll 1$, Eq. (2) degenerate into $E=D/\varepsilon$ which describes the linear dielectric behavior. When the electric field is high, $\varepsilon E/D_s\gg 1$ and Eq. (2) becomes $D=D_s$.

In the light of the work conjugated parameters: the nominal electric field and the nominal electric displacement, the electric energy density of a dielectric elastomer undergoing large deformation can be written as follows[9]:

$$V(D^{\sim})=\int_0^{D^{\sim}}E^{\sim}dD^{\sim}. \quad (3)$$

Considering equations $E^{\sim}=E\lambda_1^{-1}\lambda_2^{-1}$, $D=D^{\sim}\lambda_1^{-1}\lambda_2^{-1}$ and (3), we obtain the special electric energy of the dielectric elastomer as

$$V(\lambda_1,\lambda_2,D^{\sim})=\frac{D_sD^{\sim}\lambda_1^{-1}\lambda_2^{-1}}{2\varepsilon}\log\left(\frac{1+D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}{1-D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}\right)+\frac{D_s^2}{2\varepsilon}\log\left(1-\frac{D^{\sim 2}\lambda_1^{-2}\lambda_2^{-2}}{D_s^2}\right). \quad (4)$$

C. Large deformation, electromechanical instability, and snap-through instability

1. Constitutive law

After comprehensive consideration of Eqs. (1) and (4), we built the free energy of incompressible dielectric elastomer soft active materials as follows:

$$W(\lambda_1,\lambda_2,D^{\sim})=\frac{\mu}{2}\left[-\alpha J_{lim}\log\left(1-\frac{J_1}{J_{lim}}\right)+(1-\alpha)J_2\right]+\frac{D_sD^{\sim}\lambda_1^{-1}\lambda_2^{-1}}{2\varepsilon}\log\left(\frac{1+D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}{1-D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}\right)+\frac{D_s^2}{2\varepsilon}\log\left(1-\frac{D^{\sim 2}\lambda_1^{-2}\lambda_2^{-2}}{D_s^2}\right). \quad (5)$$

Considering the relationships among nominal stress, nominal electrical field, and free energy, we get the nominal stress and nominal electric field of dielectric elastomer soft active materials as shown below.

$$s_1 = \mu\left[\frac{\alpha J_{\lim}}{J_{\lim}-J_1}(\lambda_1-\lambda_1^{-3}\lambda_2^{-2})+(1-\alpha)(-\lambda_1^{-3}+\lambda_1\lambda_2^2)\right] - \frac{D_s D^{\sim}\lambda_1^{-2}\lambda_2^{-1}}{2\varepsilon}\log\left(\frac{1+D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}{1-D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}\right), \quad (6)$$

$$s_2 = \mu\left[\frac{\alpha J_{\lim}}{J_{\lim}-J_1}(\lambda_2-\lambda_1^{-2}\lambda_2^{-3})+(1-\alpha)(-\lambda_2^{-3}+\lambda_2\lambda_1^2)\right] - \frac{D_s D^{\sim}\lambda_1^{-1}\lambda_2^{-2}}{2\varepsilon}\log\left(\frac{1+D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}{1-D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}\right), \quad (7)$$

$$\frac{E^{\sim}}{E_s} = \frac{\lambda}{2}\log\left(\frac{1+D^{\sim}\lambda/D_s}{1-D^{\sim}\lambda/D_s}\right). \quad (8)$$

Furthermore, the true stress can be expressed as

$$\sigma_1 = \mu\left[\frac{\alpha J_{\lim}}{J_{\lim}-J_1}(\lambda_1^2-\lambda_1^{-2}\lambda_2^{-2})+(1-\alpha)(-\lambda_1^{-2}+\lambda_1^2\lambda_2^2)\right] - \frac{D_s D^{\sim}\lambda_1^{-1}\lambda_2^{-1}}{2\varepsilon}\log\left(\frac{1+D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}{1-D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}\right), \quad (9)$$

$$\sigma_2 = \mu\left[\frac{\alpha J_{\lim}}{J_{\lim}-J_1}(\lambda_2^2-\lambda_1^{-2}\lambda_2^{-2})+(1-\alpha)(-\lambda_2^{-2}+\lambda_1^2\lambda_2^2)\right] - \frac{D_s D^{\sim}\lambda_1^{-1}\lambda_2^{-1}}{2\varepsilon}\log\left(\frac{1+D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}{1-D^{\sim}\lambda_1^{-1}\lambda_2^{-1}/D_s}\right). \quad (10)$$

We now investigate the effect of the strain-stiffening on the electromechanical stability and snap-through stability of dielectric elastomers experiencing polarization saturation. The equal-biaxial planar stretch process stands for the physical process of using mechanical force to make the dielectric elastomer pre-stretched and then apply electric field to it. Under the equal-biaxial condition of a dielectric elastomer, in which $\lambda_1 = \lambda_2 = \lambda$, the free energy can be simplified as

$$W(\lambda, D^{\sim}) = \frac{\mu}{2}\left[-\alpha J_{lim}\log\left(1-\frac{2\lambda^2+\lambda^{-4}-3}{J_{lim}}\right) + (1-\alpha)(2\lambda^{-2}+\lambda^4-3)\right] + \frac{D_s D^{\sim}\lambda^{-2}}{2\varepsilon}\log\left(\frac{1+D^{\sim}\lambda^{-2}/D_s}{1-D^{\sim}\lambda^{-2}/D_s}\right) + \frac{D_s^2}{2\varepsilon}\log\left(1-\frac{D^{\sim 2}\lambda^{-4}}{D_s^2}\right). \quad (11)$$

The non-dimensional nominal electric field and nominal stress are

$$\frac{s}{\mu} = \frac{2\alpha J_{\lim}}{J_{\lim}-(2\lambda^2+\lambda^{-4}-3)}(\lambda-\lambda^{-5})+2(1-\alpha)(\lambda^3-\lambda^{-3}) - \frac{D_s}{\sqrt{\mu\varepsilon}}\frac{D^{\sim}\lambda^{-3}}{\sqrt{\mu\varepsilon}}\log\left(\frac{1+\frac{D^{\sim}\lambda^{-2}}{\sqrt{\mu\varepsilon}}\Big/\frac{D_s}{\sqrt{\mu\varepsilon}}}{1-\frac{D^{\sim}\lambda^{-2}}{\sqrt{\mu\varepsilon}}\Big/\frac{D_s}{\sqrt{\mu\varepsilon}}}\right) \quad (12)$$

and

$$\frac{E^{\sim}}{E_s} = \frac{\lambda}{2}\log\left(\frac{1+D^{\sim}\lambda/D_s}{1-D^{\sim}\lambda/D_s}\right), \quad (13)$$

respectively.

We can normalize the electric field as $\frac{E^{\sim}}{\sqrt{\mu/\varepsilon}}$, and the force as $\frac{s}{\mu}$. The extension limit of polymer chains is represented by the dimensionless parameter $J_{\lim}$, and the polarization saturation of dipoles is denoted by the dimensionless parameter $\frac{D_s}{\sqrt{\mu\varepsilon}}$.

According to Eq. (13), we can obtain that $D^{\sim} = \left[\frac{\exp(2E^{\sim}\lambda^{-1}/E_s)-1}{\exp(2E^{\sim}\lambda^{-1}/E_s)+1}\right]D_s\lambda^{-1}$, $\frac{2E^{\sim}\lambda^{-1}}{E_s} = \log\left(\frac{1+D^{\sim}\lambda/D_s}{1-D^{\sim}\lambda/D_s}\right)$, and deduce the governing equations regarding the nominal electric field and nominal electric displacement in the equilibrium state

$$\frac{s}{\mu} = \frac{2\alpha J_{\lim}}{J_{\lim}-(2\lambda^2+\lambda^{-4}-3)}(\lambda-\lambda^{-5})+2(1-\alpha)(\lambda^3-\lambda^{-3}) - 2\frac{D_s}{\sqrt{\mu\varepsilon}}\left[\frac{\exp\left(2\frac{E^{\sim}}{\sqrt{\mu/\varepsilon}}\lambda^2\Big/\frac{D_s}{\sqrt{\mu\varepsilon}}\right)-1}{\exp\left(2\frac{E^{\sim}}{\sqrt{\mu/\varepsilon}}\lambda^2\Big/\frac{D_s}{\sqrt{\mu\varepsilon}}\right)+1}\right]\frac{E^{\sim}}{\sqrt{\mu/\varepsilon}}\lambda. \quad (14)$$

Equation (13) reveals the relation between the dimensionless nominal electric field and the nominal displacement of a dielectric elastomer undergoing polarization saturation with the effect of strain-stiffening considered. Equation (14) reveals the relation between the dimensionless nominal electric field and the stretch of a dielectric elastomer under the same conditions.

Figure 4 describes the influence of different material parameters α on the deformation of soft material dielectric elastomers. In this paper, we ignore the influence of elastomers' pre-stress and do not consider their polarization saturation, which means $\frac{s}{\mu} = 0$ and $D_s \to \infty$. Also neglected is elastomers' strain-stiffening. It means that $J_{\lim} \to \infty$. As shown in Figure 4, with the deformation increasing, the nominal electrical field also increases before reaching the critical value. Then the curve either goes down or tends to be stable. For $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{1}{4}$ and $\alpha = \frac{1}{8}$, the critical nominal electrical fields of soft material dielectric elastomers, $\frac{E^{\sim}_{\max}}{\sqrt{\mu/\varepsilon}}$s, are 0.687, 0.812, 0.896, 0.945 and values of critical deformation, λ_cs, are 1.26, 1.48, 1.78, 2.15, respectively. Compared with previous investigation, the material constant ratio $k = \frac{1-\alpha}{\alpha}$. For $\alpha = 1, \frac{1}{2}, \frac{1}{4}$, and $\frac{1}{8}$, the material constant ratio will be $k = 0$, 1, 3, 7, respectively.

Figure 5 shows the relationships between $\frac{E^{\sim}}{\sqrt{\mu/\varepsilon}}$ and λ when $D_s \to \infty$, $J_{\lim} \to \infty$, $\frac{s}{\mu} = 0$, and $\alpha = \frac{1}{2}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$, respectively. As $\frac{s}{\mu}$ increases, the critical nominal electrical field decreases while the critical deformation increases. Now we list several critical values. When $\frac{s}{\mu} = 0, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$, soft material dielectric elastomers' critical nominal electrical fields will be $0.812\sqrt{\mu/\varepsilon}$, $0.800\sqrt{\mu/\varepsilon}$, $0.790\sqrt{\mu/\varepsilon}$, $0.771\sqrt{\mu/\varepsilon}$, and the values of critical deformation will be 1.48, 1.52, 1.57, 1.68, respectively.

The electric field-stretch curves with different $\frac{D_s}{\sqrt{\mu\varepsilon}}$s are shown in Figure 6. Here, $J_{\lim} \to \infty$, $\frac{s}{\mu} = 0$. In Figures 6(a)–6(d),

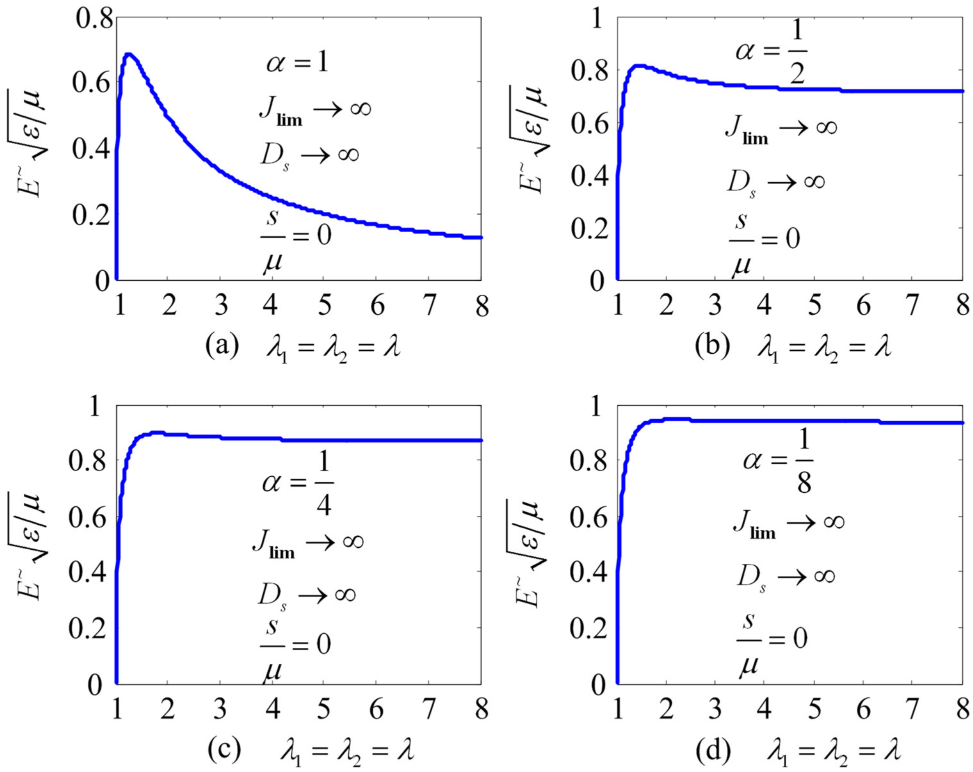

FIG. 4. The relation between nominal electric field and the stretch of the dielectric elastomer soft materials under specialized load condition, namely, $\lambda_1 = \lambda_2 = \lambda$, $\frac{s}{\mu} = 0$, and specialized material limit condition $J_{\lim} \to \infty$, $D_s \to \infty$. (a) $\alpha = 1$, (b) $\alpha = \frac{1}{2}$, (c) $\alpha = \frac{1}{4}$, (d) $\alpha = \frac{1}{8}$.

$\alpha = 1$while in Figures 6(e)–6(h), $\alpha = \frac{1}{2}$. The effect of polarization saturation is evaluated by inspecting the state equations (12) and (14). In Figures 6(a)–6(d), the local maximum is eliminated when $D_s/\sqrt{\mu\varepsilon}$ is small. In consideration of $\frac{s}{\mu} = 0$ and $J_{\lim} \to \infty$in Figures 6(a)–6(d), we note that the electric field approaches a limiting value $E^{\sim} = \mu/D_s$ as $\lambda \to \infty$. From Figures 6(e)–6(h) we can see that when $\frac{D_s}{\sqrt{\mu\varepsilon}}$ is small enough, the nominal electric field increases monotonously with the stretch, avoiding any local maximum value and the dielectric elastomer can therefore approach its stable state.

Figure 7 depicts the voltage-stretch curves of dielectric elastomers with $D_s \to \infty$, $\frac{s}{\mu} = 0$, $\alpha = \frac{1}{2}$ and several values of $J_{\lim}$. In Figures 7(a)–7(c), when $D_s \to \infty$ and $J_{\lim}$ has a finite value, the nominal electric field attains a local maximum value first. Then, the dielectric elastomer goes through the snap instability with the voltage increasing. In Figure 7(d),

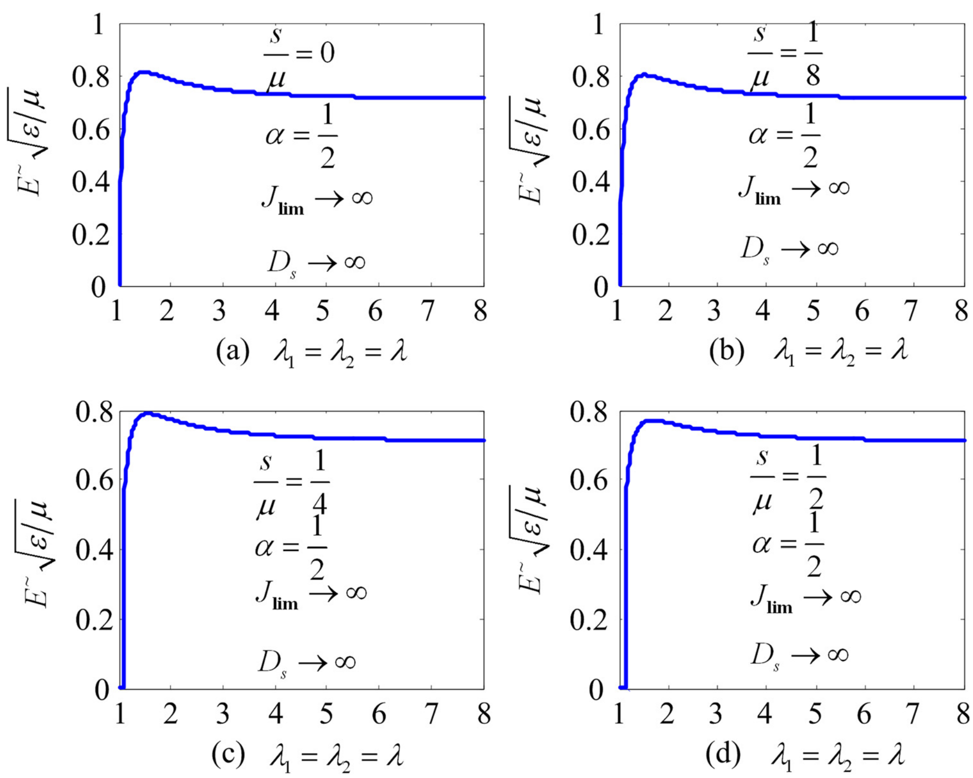

FIG. 5. The relation between nominal electric field and the stretch of the dielectric elastomer soft materials under specialized load condition, namely, $\lambda_1 = \lambda_2 = \lambda$, material constant $\alpha = \frac{1}{2}$, and specialized material limit condition $J_{\lim} \to \infty$, $D_s \to \infty$. (a) $\frac{s}{\mu} = 0$, (b) $\frac{s}{\mu} = \frac{1}{8}$, (c) $\frac{s}{\mu} = \frac{1}{4}$, (d) $\frac{s}{\mu} = \frac{1}{2}$.

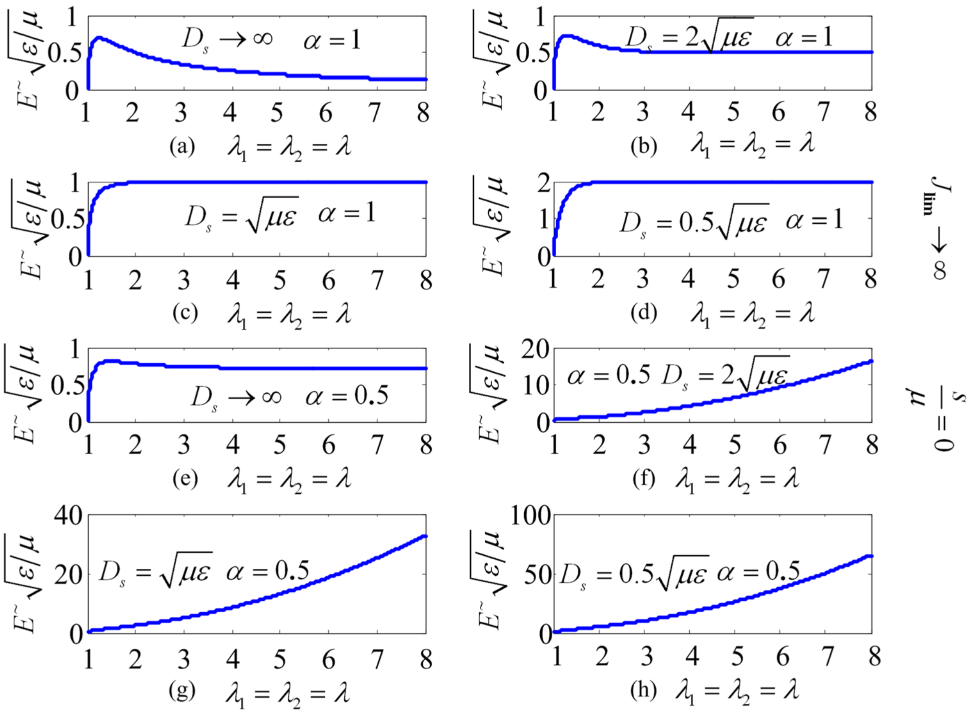

FIG. 6. The relation between nominal electric field and the stretch of the dielectric elastomer soft materials under specialized load condition, namely, $\lambda_1 = \lambda_2 = \lambda$, and specialized material limit condition $J_{\mathrm{lim}} \to \infty$, $\frac{s}{\mu} = 0$. (a) $\alpha = 1$, $D_s \to \infty$ (b) $\alpha = 1$, $\frac{D_s}{\sqrt{\mu\varepsilon}} = 2$, (c) $\alpha = 1$, $\frac{D_s}{\sqrt{\mu\varepsilon}} = 1$ (d) $\alpha = 1$, $\frac{D_s}{\sqrt{\mu\varepsilon}} = \frac{1}{2}$, (e) $\alpha = \frac{1}{2}$, $D_s \to \infty$, (f) $\alpha = \frac{1}{2}$, $\frac{D_s}{\sqrt{\mu\varepsilon}} = 2$, (g) $\alpha = \frac{1}{2}$, $\frac{D_s}{\sqrt{\mu\varepsilon}} = 1$, (h) $\alpha = \frac{1}{2}$, $\frac{D_s}{\sqrt{\mu\varepsilon}} = \frac{1}{2}$.

when $J_{\mathrm{lim}} \to \infty$ and $D_s \to \infty$, the dielectric elastomer undergoes the pull-in electromechanical instability.

VI. CONCLUDING REMARKS

We depicted electromechanical response of deformable dielectrics in the voltage-deformation plane. The effects of material limits, including polarization saturation, strain stiffening, and breakdown voltage, as well as of pre-stretch are also investigated. We found that the actuation deformation of stiff dielectrics is small whose potential is determined by the breakdown voltage, while soft dielectrics, characterized by dielectric elastomes, have dramatically larger actuation deformation which is limited by breakdown voltage, electromechanical instability, and snap-through instability in a synergic manner. Furthermore, we can obtain the theoretical prediction of maximum actuation deformation with respect to certain degrees of pre-stretch for specific deformable dielectrics.

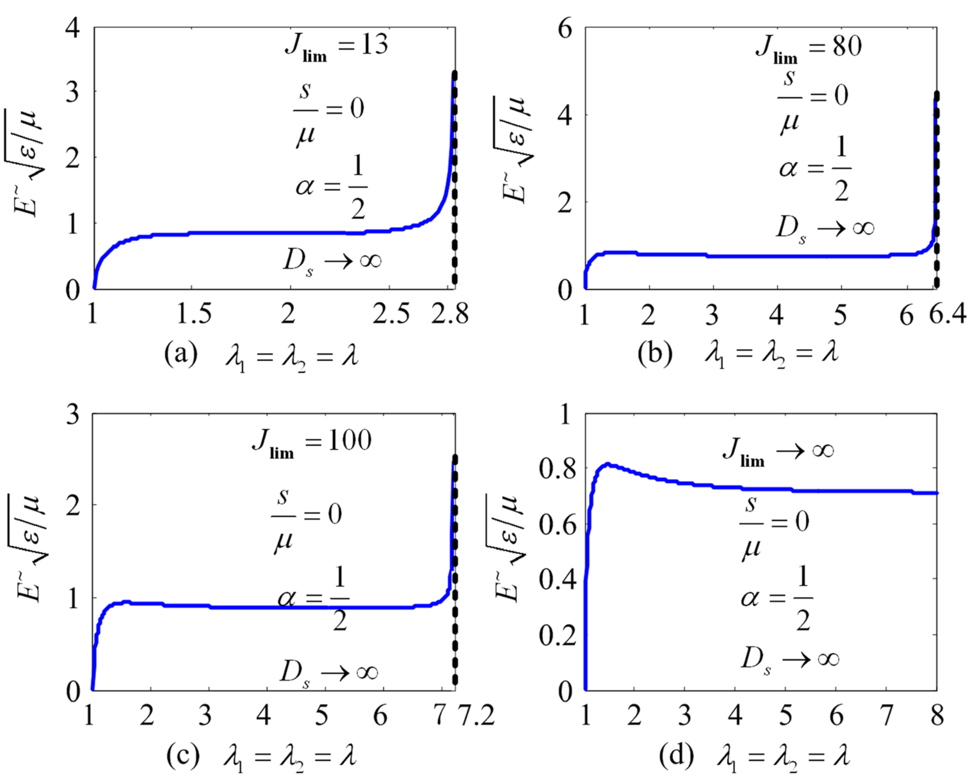

FIG. 7. The relation between nominal electric field and the stretch of the dielectric elastomer soft materials under specialized load condition, namely, $\lambda_1 = \lambda_2 = \lambda$, $\frac{s}{\mu} = 0$ material constant $\alpha = \frac{1}{2}$, and specialized material limit condition $D_s \to \infty$. (a) $J_{\mathrm{lim}} = 13$, (b) $J_{\mathrm{lim}} = 80$, (c) $J_{\mathrm{lim}} = 100$, (d) $J_{\mathrm{lim}} \to \infty$.

ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China (Grant No. 11102052) and the Fundamental Research Funds for the Central Universities (Grant No. HIT.NSRIF.2013030).

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