SCIENCE CHINA
Physics, Mechanics & Astronomy

• Article •

January 2012 Vol.55 No.1: 94–101
doi: 10.1007/s11433-011-4537-0

Effect of mechanical force field on the electromechanical stability of dielectric elastomers

ZHANG Zhen[1], LIU LiWu[1,2], LIU YanJu[2], LENG JinSong[1*] & DU ShanYi[1]

[1] *Centre for Composite Materials, Science Park of Harbin Institute of Technology, Harbin 150080, China;*
[2] *Department of Astronautical Science and Mechanics, Harbin Institute of Technology, Harbin 150001, China*

Received May 7, 2011; accepted September 20, 2011; published online October 26, 2011

To study the electromechanical stability of dielectric elastomer subjected to a mechanical force field, we use free energy functions of variable forms to analyze the mechanical performance of dielectric elastomer. The relation among critical nominal electric field, critical true electric field, nominal stress and mechanical force field is derived. These calculations agree well with the experimental results. The results can help us better understand the stability conditions of dielectric elastomers and furthermore guide the design and manufacture of sensors and actuators based on dielectric elastomers.

dielectric elastomer, mechanical force field, stability analysis

PACS number(s): 46.05.+6, 77.55.+f, 81.05.Lg

Citation: Zhang Z, Liu L W, Liu Y J, et al. Effect of mechanical force field on the electromechanical stability of dielectric elastomers. Sci China-Phys Mech Astron, 2012, 55: 94–101, doi: 10.1007/s11433-011-4537-0

Nomenclature

L_1, L_2, L_3	original sides of dielectric elastomer
F_1, F_2	mechanical forces
U	electrical voltage
Q	electric charge
λ_1, λ_2, λ_3	stretch ratio
l_1, l_2, l_3	dielectric elastomer sides after deformation
s_1, s_2	nominal stress
$\tilde{E}$	nominal electric field
$\tilde{D}$	nominal displacement
σ_1, σ_2	true stress
E	true electric field
D	true displacement
$\tilde{E}_{\max}$	critical nominal electric field
h_c	critical thickness strain
λ_c	critical stretch ratio
$E_{\max}$	critical true electric field
ε	relative permittivity
C_1, C_2, μ_p, α_p	material constants
k	material constant ratio

The electromechanical stability theory of dielectric elastomers was first proposed by Suo and Zhao [1–5]. Recent years have seen extensive and in-depth studies of the stability analysis of dielectric elastomer [6–28]. When a voltage is applied on the dielectric elastomer film, the voltage will cause the film to become thinner [6–16, 29–38]. As a result, the same voltage will induce a higher electric field and further cause the dielectric elastomer film to become thinner. When the electric field reaches the critical breakdown electric field, the resulting breakdown will occur in the dielectric elastomer. This process is called electromechanical instability or pull-in instability [3,4].

Zhao and Suo [1,2] proposed that free energy function of any form can be used to analyze the electromechanical stability of dielectric elastomer. As a special case, the elastic strain energy function with one material constant was used

*Corresponding author (email: Lengjs@hit.edu.cn)

to analyze the stability of an ideal elastic elastomer under equal biaxial pre-stresses and unequal biaxial pre-stresses. The results illustrated the relation between the nominal electric displacement and the nominal electric field. They showed theoretically that pre-stretch can enhance the stability of dielectric elastomer, which agrees with the experimental tests.

Norrisa applied the Ogden elastic strain energy model to analyze the stability of the dielectric elastomer [7]. The relation among the critical electric field, the nominal strain and the pre-stretch of dielectric elastomer was obtained accurately. Simultaneously, as a special case, the Neo-Hookean elastic strain energy model, which was a simplified version of the Ogden model, was introduced to give more concise but accurate results.

Further research on the stability of Neo-Hookean type dielectric elastomer was done by Díaz-Calleja's group [8]. The Hessian matrix of dielectric elastomer under two special conditions was deduced. Furthermore, the stable domain and unstable domain of dielectric elastomer were described. These results can help us understand the stability performance of Neo-hooken type dielectric elastomer more thoroughly.

Ref. [10] applied the elastic strain energy function with two material constants to analyze the stability performance of dielectric elastomers. The introduced material constant ratio k could help us analyze the stability of various dielectric elastomers. The relation between the nominal electric displacement and nominal electric field of various dielectric elastomers was derived.

In this paper, variable free energy functions are applied to analyze the mechanical performance of dielectric elastomer subjected to a mechanical force field. The relation among critical nominal electric field, critical true electric field, nominal stress and mechanical force field is derived, which agrees well with the experimental results.

1 Stability parameters of electromechanical coupling system

As is shown in Figure 1, in the referenced state, the membrane of dielectric elastomer is of sides L_1, L_2, and L_3. When the dielectric elastomer is subject to mechanical forces F_1, L_2, and to electrical voltage U, the three sides of the membrane deform to l_1, l_2 and l_3, and electric charge Q flows through the external circuit from one electrode to another. Here λ_i, $i=1, 2, 3$ represents the principal stretch ratio after deformation, and s_1, s_2 are the nominal stress, which can be derived in the undeformed state by dividing the pre-stretch force by the area, i.e., $s_1=F_1/L_2L_3$, $s_2=F_2/L_1L_3$. The dielectric elastomer is taken to be incompressible, so that $\lambda_1=1/\lambda_1\lambda_2$. Similarly, the nominal electric field $\tilde{E}=U/L_e$ is defined as the voltage divided by the original thickness, and the nominal electric displacement $\tilde{D}=Q/L_1L_2$ is defined as the charge divided by the area before deformation. The corresponding true electric field can be obtained by dividing the electric voltage U by the thickness in the deformed state, i.e., $E=U/\lambda_3L_3$, and also the corresponding true electric displacement is $D=Q/\lambda_1L_1\lambda_2L_2$.

In this paper, the effect of mechanical force field on the electromechanical stability of dielectric elastomers is studied. Stability performance parameters are defined and the mathematical expressions of those parameters are derived. Here we define the critical nominal electric field $\tilde{E}_{\max}$, the critical true electric field $E_{\max}$, the nominal stress s, the true stress σ, the critical stretch ratio λ_C, the critical thickness strain $h_\varepsilon=1-1/\lambda_C^2$ as the stability performance parameters, where the critical quantities and the true quantities represent respectively the different physical performance from the stable states to the unstable states, while the ratio variables denote the deformation comportment.

2 The mathematical expression of system stability parameters

To the electromechanical coupling system of the dielectric elastomer, elastic strain energy function (mechanical force field) and the electric energy destiny function (electric field) can be coupled. Then the free energy function of the coupling system is obtained [15]:

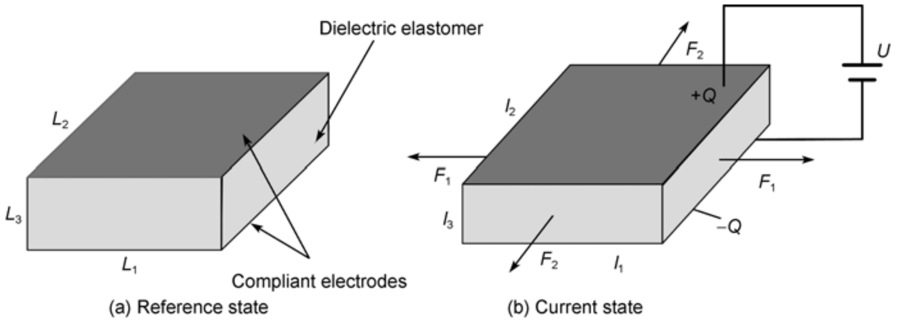

Figure 1 The dielectric elastomer is sandwiched between two compliant electrodes. (a) In the reference state, the dielectric is subjected to neither forces nor voltage; (b) in the current state, subject to forces and voltage, the dielectric elastomer deforms.

$$W(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},D^{\sim})=U(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1})+V(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},D^{\sim}), \quad (1)$$

where $W(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},D^{\sim})$ denotes the free energy function of dielectric elastomer electromechanical coupling system, $U(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1})$ represents the elastic strain energy function, and $V(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},D^{\sim})$ reveals the electric energy destiny function. The Hessian matrix is

$$H=\begin{bmatrix} U_{11}+V_{11} & U_{12}+V_{12} & V_{13} \\ U_{12}+V_{12} & U_{22}+V_{22} & V_{23} \\ V_{13} & V_{23} & V_{33} \end{bmatrix}, \quad (2)$$

where

$$U_{ij}=\frac{\partial U}{\partial\lambda_i\partial\lambda_j},\quad V_{ij}=\frac{\partial V}{\partial\lambda_i\partial\lambda_j},\quad i,j=1,2,$$

$$V_{13}=\frac{\partial V}{\partial\lambda_1\partial D^{\sim}},\quad V_{23}=\frac{\partial V}{\partial\lambda_2\partial D^{\sim}},\quad V_{33}=\frac{\partial V}{\partial D^{\sim 2}}.$$

The elastic strain energy function above represents the effect of mechanical force field on the dielectric elastomer electromechanical coupling system, and the electric energy destiny function represents the effect of electric field on the dielectric elastomer electromechanical coupling system. For the effect of mechanical force field, the electric energy destiny function can be set as a constant value. So the model of the electric energy destiny function with the dielectric constant ε invariable can be expressed as follows:

$$V(\lambda_1,\lambda_2,\lambda_1^{-1}\lambda_2^{-1},D^{\sim})=\frac{D^{\sim 2}}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \quad (3)$$

Inserting eq. (3) into eq. (2), we obtain the Hessian matrix of dielectric elastomer:

$$H=\begin{bmatrix} U_{11}+\frac{3D^{\sim 2}}{\varepsilon}\lambda_1^{-4}\lambda_2^{-2} & U_{12}+\frac{2D^{\sim 2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2D^{\sim}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} \\ U_{12}+\frac{2D^{\sim 2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & U_{22}+\frac{3D^{\sim 2}}{\varepsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2D^{\sim}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} \\ -\frac{2D^{\sim}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2D^{\sim}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix}. \quad (4)$$

And then the determinant of Hessian matrix is

$$\det(H)=\frac{U_{11}U_{12}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}-\frac{U_{11}D^{\sim 2}}{\varepsilon^2}\lambda_1^{-4}\lambda_2^{-6}+\frac{3D^{\sim 2}U_{22}}{\varepsilon^2}\lambda_1^{-6}\lambda_2^{-4}-\frac{3D^{\sim 4}}{\varepsilon^3}\lambda_1^{-8}\lambda_2^{-8}-\frac{U_{12}^{\ 2}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}$$
$$+\frac{4D^{\sim 4}}{\varepsilon^3}\lambda_1^{-8}\lambda_2^{-8}+\frac{4U_{12}}{\varepsilon^2}\lambda_1^{-5}\lambda_2^{-5}+\frac{8D^{\sim 4}}{\varepsilon^3}\lambda_1^{-8}\lambda_2^{-8}-\frac{4D^{\sim 2}U_{22}}{\varepsilon^2}\lambda_1^{-6}\lambda_2^{-4}-\frac{12D^{\sim 4}}{\varepsilon^3}\lambda_1^{-8}\lambda_2^{-8}. \quad (5)$$

For such a dielectric elastomer electromechanical coupling system, we have $D^{\sim}=E^{\sim}\varepsilon\lambda_1^2\lambda_2^2$ and $E=E^{\sim}\lambda_1\lambda_2$, where $E^{\sim}$ represents the nominal electric field, and E denotes the true electric field. Inserting them into eq. (5), the accurate relation of critical nominal breakdown electric field $E^{\sim}_{\max}$, critical true breakdown electric field $E_{\max}$, nominal strain and mechanical force field on the dielectric elastomer are obtained as follows:

$$\varepsilon E^{\sim 2}_{\max}=\frac{1}{6}\Big[4U_{12}\lambda_1^{-1}\lambda_2^{-1}-\lambda_2^{-2}U_{11}-\lambda_1^{-2}U_{22}+\sqrt{(\lambda_2^{-2}U_{11}+\lambda_1^{-2}U_{22}-4U_{12}\lambda_1^{-1}\lambda_2^{-1})^2+12\lambda_1^{-2}\lambda_2^{-2}(U_{11}U_{22}-U_{12}^2)}\Big], \quad (6)$$

$$\varepsilon E^{2}_{\max}=\frac{1}{6}\Big[4U_{12}\lambda_1\lambda_2-\lambda_1^{2}U_{11}-\lambda_2^{2}U_{22}+\sqrt{(\lambda_1^{2}U_{11}+\lambda_2^{2}U_{22}-4U_{12}\lambda_1\lambda_2)^2+12\lambda_1^{2}\lambda_2^{2}(U_{11}U_{22}-U_{12}^2)}\Big], \quad (7)$$

$$s_j=U_j-\lambda_j^{-1}\varepsilon E^2, \quad (8)$$

$$\sigma_j=\lambda_j U_j-\varepsilon E^2_{\max}, \quad (9)$$

j=1, 2. Let $s_j=0$, and the maximum stretch λ_{C} can be calculated. Inserting this result into eqs. (6) and (7), we can get the critical nominal electric field $E^{\sim}_{\max}$, and the critical true electric field $E_{\max}$. Meanwhile we can get the critical thickness strain $h_\varepsilon=1-1/\lambda_{\mathrm{C}}^2$, which represents the maximum deformation of dielectric elastomer under the coupling of electric field and mechanical field.

To simplify the above relations, we take into account the special condition of equal biaxial pre-stretch of dielectric elastomer, namely, $\lambda_1=\lambda_2=\lambda$, then $U_{11}=U_{22}$, and inserting them into eqs. (6)–(9), we obtain:

$$\varepsilon E^{\sim 2}_{\max}=\frac{\lambda^{-2}}{3}(U_{11}+U_{12}), \quad (10)$$

$$\varepsilon E^{2}_{\max}=\frac{\lambda^{2}}{3}(U_{11}+U_{12}), \quad (11)$$

$$U_1-\frac{\lambda}{3}(U_{11}+U_{12})=s, \quad (12)$$

$$U_1\lambda-\frac{\lambda^2}{3}(U_{11}+U_{12})=\sigma, \quad (13)$$

$$h_\varepsilon = 1 - 1/\lambda_c^2. \tag{14}$$

Eqs. (10)–(14) denote the accurate relations of critical nominal electric field, critical true electric field, nominal strain and elastic strain energy function (effect of mechanical force field) under the condition of equal biaxial pre-stretch.

3 The effect of mechanical force field

The dielectric elastomer electromechanical stability is analyzed in detail in this part when the dielectric elastomer is subjected to a mechanical force field. When different elastic strain energy functions are introduced, the explicit expression of the dielectric elastomer performance parameters is obtained under the condition of equal biaxial pre-stretch. By introducing the model of elastic strain energy function with one material constant, the free energy function can be written as:

$$W(\lambda_1,\lambda_2,\tilde{D}) = \frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3) + \frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \tag{15}$$

$$s_1 = \frac{\partial W}{\partial \lambda_1} = \mu(\lambda_1 - \lambda_1^{-3}\lambda_2^{-2}) - \frac{\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{16}$$

$$s_2 = \frac{\partial W}{\partial \lambda_2} = \mu(\lambda_2 - \lambda_2^{-3}\lambda_1^{-2}) - \frac{\tilde{D}^2}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2}, \tag{17}$$

$$\tilde{E} = \frac{\partial W}{\partial \tilde{D}} = \frac{\tilde{D}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{18}$$

Inserting the above equations into eq. (4), the Hessian matrix of dielectric elastomer is

$$H = \begin{bmatrix} \mu(1+3\lambda_1^{-4}\lambda_2^{-2}) + \frac{3\tilde{D}^2}{\varepsilon}\lambda_1^{-4}\lambda_2^{-2} & 2\mu\lambda_1^{-3}\lambda_2^{-3} + \frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} \\ 2\mu\lambda_1^{-3}\lambda_2^{-3} + \frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & \mu(1+3\lambda_2^{-4}\lambda_1^{-2}) + \frac{3\tilde{D}^2}{\varepsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} \\ -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix}. \tag{19}$$

The special condition of dielectric elastomer loading is taken into account, namely $\lambda_1 = \lambda_2 = \lambda$. We have $U_{11} = U_{22}$, and inserting it into eqs. (10)–(13), we get:

$$\varepsilon \tilde{E}_{\max}^2 = \frac{\mu}{3}(\lambda^{-2} + 5\lambda^{-8}), \tag{20}$$

$$\varepsilon E_{\max}^2 = \frac{\mu}{3}(\lambda^2 + 5\lambda^{-4}), \tag{21}$$

$$s = \frac{2\mu}{3}(\lambda - 4\lambda^{-5}), \tag{22}$$

$$\sigma = \frac{2\mu}{3}(\lambda^2 - 4\lambda^{-4}). \tag{23}$$

For eq. (22), let s=0. The stretch λ reaches its critical value $\lambda_{\max}$, then $\lambda_{\max} = 1.26$. By inserting $\lambda_{\max} = 1.26$ into eqs. (20) and (21), critical nominal electric field and critical true electric field are obtained, namely $\tilde{E}_{\max} = 0.69\sqrt{\mu/\varepsilon}$, $E_{\max} = 1.09\sqrt{\mu/\varepsilon}$. Inserting them into eq. (14), then the value of the critical thickness strain h_ε is about 37%. Clearly, the results coincide well with Suo's conclusion [1].

Figure 2 gives the relations between electromechanical stability parameters and the stretch of dielectric elastomer, including critical nominal electric field, critical true electric field, critical nominal stress, and critical true stress.

By introducing the model of elastic strain energy function with two material constants, the free energy function can be expressed as follows:

$$W(\lambda_1,\lambda_2,\tilde{D}) = \frac{C_1}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3) + \frac{C_2}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3) + \frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \tag{24}$$

$$s_1 = \frac{\partial W}{\partial \lambda_1} = C_1(\lambda_1 - \lambda_1^{-3}\lambda_2^{-2}) + C_2(-\lambda_1^{-3} + \lambda_1\lambda_2^2) - \frac{\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{25}$$

$$s_2 = \frac{\partial W}{\partial \lambda_2} = C_1(\lambda_2 - \lambda_2^{-3}\lambda_1^{-2}) + C_2(-\lambda_2^{-3} + \lambda_2\lambda_1^2) - \frac{\tilde{D}^2}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2}, \tag{26}$$

$$\tilde{E} = \frac{\partial W}{\partial \tilde{D}} = \frac{\tilde{D}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{27}$$

Substituting it into eq. (4), the Hessian matrix of dielectric elastomer can be written as follows:

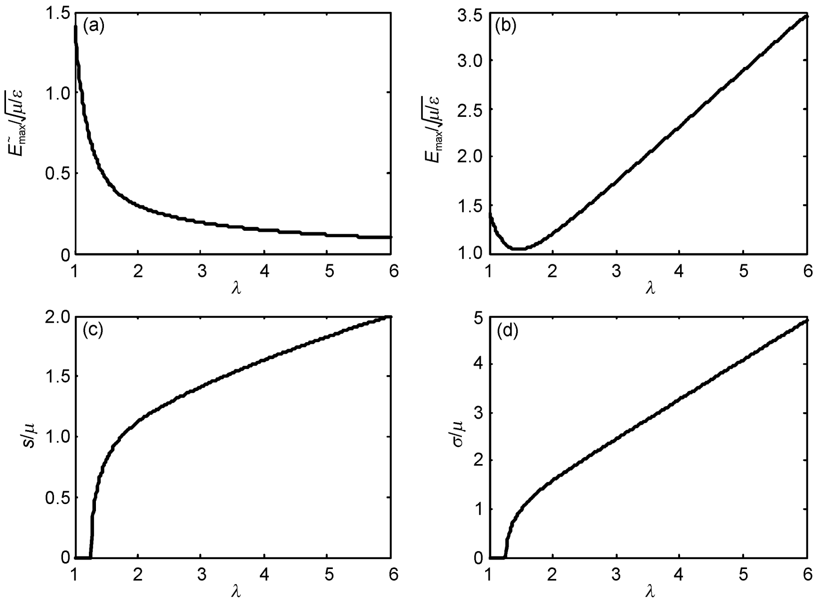

Figure 2 Relations between electromechanical stability parameters and the stretch of the dielectric elastomer. (a) Critical nominal electric field; (b) critical true electric field; (c) critical nominal stress; (d) critical true stress.

$$H=\begin{bmatrix} C_1(1+3\lambda_1^{-4}\lambda_2^{-2})+C_2(3\lambda_1^{-4}+\lambda_2^{2})+\frac{3\tilde{D}^2}{\varepsilon}\lambda_1^{-4}\lambda_2^{-2} & 2C_1\lambda_1^{-3}\lambda_2^{-3}+2C_2\lambda_1\lambda_2+\frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} \\ 2C_1\lambda_1^{-3}\lambda_2^{-3}+2C_2\lambda_1\lambda_2+\frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & C_1(1+3\lambda_2^{-4}\lambda_1^{-2})+C_2(3\lambda_2^{-4}+\lambda_1^{2})+\frac{3\tilde{D}^2}{\varepsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} \\ -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix}. \quad (28)$$

Considering the special condition of dielectric elastomer loading, namely $\lambda_1=\lambda_2=\lambda$, then $U_{11}=U_{22}$, and substituting it into eqs. (10)–(14), we have:

$$\varepsilon \tilde{E}_{max}^2=\frac{C_1}{3}(\lambda^{-2}+5\lambda^{-8})+C_2(\lambda^{-6}+1), \quad (29)$$

$$\varepsilon E_{max}^2=\frac{C_1}{3}(\lambda^{2}+5\lambda^{-4})+C_2(\lambda^{-2}+\lambda^{4}), \quad (30)$$

$$s=\frac{2C_1}{3}(\lambda-4\lambda^{-5})-2C_2\lambda^{-3}, \quad (31)$$

$$\sigma=\frac{2C_1}{3}(\lambda^{2}-4\lambda^{-4})-2C_2\lambda^{-2}. \quad (32)$$

For convenience, we introduce a dimensionless quantity k which depends on the material and the activated shape of the dielectric elastomer, where λ_1, when λ_2 is a constant, as 0, L_1, L_2, L_3. For F_1, F_2, let s=0. The critical stretch can be evaluated to be λ_c=1.48. Inserting it into eq. (29), the maximum value of the nominal electric field $\tilde{E}_{max}$ = $1.148\sqrt{C_1/\varepsilon}$ is obtained. Substituting it into eq. (30), the maximum value of the true electric field E_{max} = $2.514\sqrt{C_1/\varepsilon}$ is obtained.

For $k=1/2$, let $s=0$, and the critical stretch can be evaluated to be $\lambda_c=1.37$. Inserting it into eq. (29), the maximum value of the nominal electric field $\tilde{E}_{max}$ = $0.936\sqrt{C_1/\varepsilon}$ is obtained. Inserting it into eq. (30), the maximum value of the true electric field E_{max} = $1.757\sqrt{C_1/\varepsilon}$ is obtained.

For $k=1/4$, let $s=0$, and the critical stretch can be evaluated to be $\lambda_c=1.32$. Inserting it into eq. (29), the maximum value of the nominal electric field $\tilde{E}_{max}$ = $0.817\sqrt{C_1/\varepsilon}$ is obtained. Inserting it into eq. (30), the maximum value of the true electric field E_{max} = $1.423\sqrt{C_1/\varepsilon}$ is obtained.

For $k=1/5$, let $s=0$, and the critical stretch can be evaluated to be $\lambda_c=1.32$. Inserting it into eq. (29), the maximum value of the nominal electric field $\tilde{E}_{max}$ = $0.792\sqrt{C_1/\varepsilon}$ is obtained. Inserting it into eq. (30), the

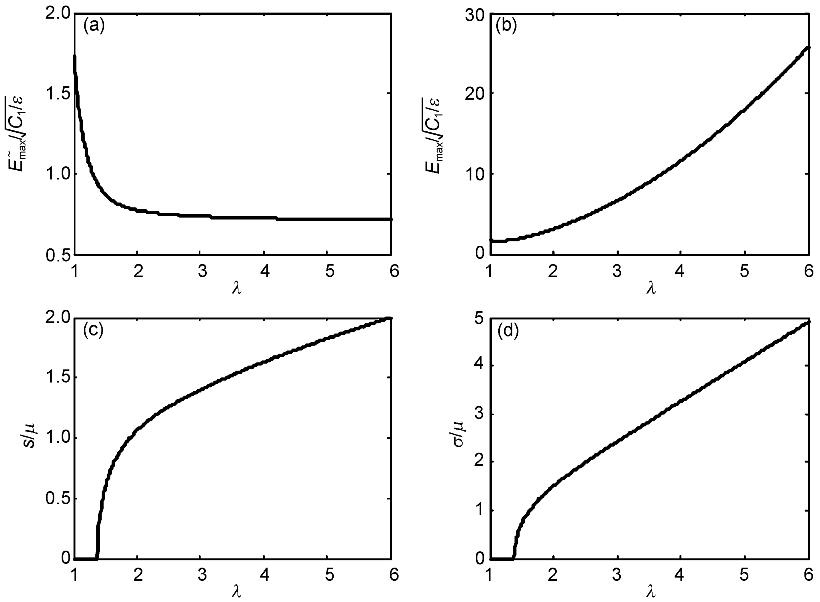

Figure 3 Relations between stability parameters and the stretch when *k*=0.5. (a) Critical nominal electric field; (b) critical true electric field; (c) critical nominal stress; (d) critical true stress.

maximum value of the true electric field $E_{\max}=1.338\sqrt{C_1/\varepsilon}$ is obtained. The results correspond to Liu's conclusion [5].

Figure 3 shows the relations between electromechanical stability parameters (critical nominal electric field, critical true electric field, critical nominal stress and critical true stress) and the stretch of dielectric elastomer under equal biaxial stresses and material constant ratio *k*=1/2.

For the model of elastic strain energy function with various material constants, the special condition of dielectric elastomer is taken into account, namely $\lambda_1=\lambda_2=\lambda$, then $U_{11}=U_{22}$. Inserting it into eqs. (6)–(8), we get

$$W(\lambda_1,\lambda_2,\tilde{D})=\sum_{p=1}^{N}\frac{\mu_p}{\alpha_p}(\lambda_1^{\alpha_p}+\lambda_2^{\alpha_p}+\lambda_1^{-\alpha_p}\lambda_2^{-\alpha_p}-3)+\frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \quad (33)$$

$$s_1=\frac{\partial W}{\partial\lambda_1}=\sum_{p=1}^{N}\mu_p(\lambda_1^{\alpha_p-1}-\lambda_1^{-\alpha_p-1}\lambda_2^{-\alpha_p})-\frac{\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \quad (34)$$

$$s_2=\frac{\partial W}{\partial\lambda_2}=\sum_{p=1}^{N}\mu_p(\lambda_2^{\alpha_p-1}-\lambda_2^{-\alpha_p-1}\lambda_1^{-\alpha_p})-\frac{\tilde{D}^2}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2}, \quad (35)$$

$$\tilde{E}=\frac{\partial W}{\partial\tilde{D}}=\frac{\tilde{D}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \quad (36)$$

Substituting eq. (33) into eq. (4), the Hessian matrix of dielectric elastomer is

$$H=\begin{bmatrix}\sum_{p=1}^{N}\mu_p[(\alpha_p-1)\lambda_1^{\alpha_p-2}+(\alpha_p+1)\lambda_1^{-\alpha_p-2}\lambda_2^{-\alpha_p}]+\frac{3\tilde{D}^2}{\varepsilon}\lambda_1^{-4}\lambda_2^{-2} & \sum_{p=1}^{N}\mu_p\alpha_p\lambda_1^{-\alpha_p-1}\lambda_2^{-\alpha_p-1}+\frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}\\ \sum_{p=1}^{N}\mu_p\alpha_p\lambda_1^{-\alpha_p-1}\lambda_2^{-\alpha_p-1}+\frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & \sum_{p=1}^{N}\mu_p[(\alpha_p-1)\lambda_2^{\alpha_p-2}+(\alpha_p+1)\lambda_2^{-\alpha_p-2}\lambda_1^{-\alpha_p}]+\frac{3\tilde{D}^2}{\varepsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2}\\ -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}\end{bmatrix}, \quad (37)$$

$$\varepsilon\tilde{E}_{\max}^2=\frac{1}{3}\sum_{p=1}^{N}\mu_p[(\alpha_p-1)\lambda^{\alpha_p-4}+(2\alpha_p+1)\lambda^{-2\alpha_p-4}], \quad (38)$$

$$\varepsilon E_{\max}^2=\frac{1}{3}\sum_{p=1}^{N}\mu_p[(\alpha_p-1)\lambda^{\alpha_p}+(2\alpha_p+1)\lambda^{-2\alpha_p}], \quad (39)$$

$$\frac{1}{3\lambda}\sum_{p=1}^{N}\mu_p[(4-\alpha_p)\lambda^{\alpha_p}-(4+2\alpha_p)\lambda^{-2\alpha_p}]=s, \quad (40)$$

$$\frac{1}{3}\sum_{p=1}^{N}\mu_p[(4-\alpha_p)\lambda^{\alpha_p}-(4+2\alpha_p)\lambda^{-2\alpha_p}]=\sigma. \quad (41)$$

For eq. (40), let s=0, and the critical stretch ratio λ_c is obtained. Inserting it into eqs. (38) and (39), then the explicit expressions of critical nominal electric and critical true electric field can be obtained. The Neo-Hookean model and the Mooney-Rivlin model can be regarded as the special case of the Ogden elastic strain energy function with various material constants. The following part is to simplify the Ogden model, and apply the simplified form to evaluate the critical nominal electric field, the critical true electric field and the nominal stress, then compare and validate them with the results obtained by the above models.

Assuming N=1, $\alpha_1=\alpha$, $\mu_1=\mu$, the critical stretch can be evaluated to be

$$\lambda_c=\left(\frac{4+2\alpha}{4-\alpha}\right)^{1/3\alpha},$$

$$\varepsilon\tilde{E}_{\max}^2=\frac{1}{3}\mu\left[(\alpha-1)\left(\frac{4+2\alpha}{4-\alpha}\right)^{\frac{\alpha-4}{3\alpha}}+(2\alpha+1)\left(\frac{4+2\alpha}{4-\alpha}\right)^{\frac{-2\alpha-4}{3\alpha}}\right], \tag{42}$$

$$\varepsilon E_{\max}^2=\frac{1}{3}\mu\left[(\alpha-1)\left(\frac{4+2\alpha}{4-\alpha}\right)^{\frac{1}{3}}+(2\alpha+1)\left(\frac{4+2\alpha}{4-\alpha}\right)^{-\frac{2}{3}}\right]. \tag{43}$$

Obviously, assuming $\alpha=2$, the critical stretch $\lambda_c=1.26$. Then inserting it into eqs. (42) and (43), the critical nominal and the critical true electric field can be obtained, namely $\tilde{E}_{\max}=0.69\sqrt{\mu/\varepsilon}$, $E_{\max}=1.09\sqrt{\mu/\varepsilon}$. For comparison, the results agree well with those by using the Neo-Hookean model.

Assuming $N=2$, $\alpha_2=-\alpha_1=-2$, $\mu_2=m\mu_1$, inserting them into eq. (40), $\lambda^6+3m\lambda^2-8=0$, when m equals different values, the critical stretch ratio λ can be obtained. $m=-1$, $\lambda_c=1.48$. Inserting them into eq. (38), the maximum value of the nominal electric field $\tilde{E}_{\max}=1.148\sqrt{\mu_1/\varepsilon}$. Inserting them into eq. (39), the maximum value of the nominal electric field $E_{\max}=2.514\sqrt{\mu_1/\varepsilon}$. When m is chosen as $-1/2$, $-1/4$, $-1/5$, respectively, the corresponding critical stretch is evaluated as λ_c=1.37, 1.32, 1.30, the critical nominal electric field $\tilde{E}_{\max}=0.936\sqrt{\mu_1/\varepsilon}$, $0.817\sqrt{\mu_1/\varepsilon}$, $0.792\sqrt{\mu_1/\varepsilon}$ respectively, while the critical true electric field $E_{\max}=1.757\sqrt{\mu_1/\varepsilon}$, $1.423\sqrt{\mu_1/\varepsilon}$, $1.338\sqrt{\mu_1/\varepsilon}$ respectively. Let $C_2=-\mu_2$, $C_1=\mu_1$, $k=-m$. The evaluated result coincides with the result when the Mooney-Rivlin model is used.

4 Conclusions

This paper has obtained the accurate relation among critical nominal, critical true electric field, nominal stress and different strain energy functions in a dielectric elastomer electromechanical coupling system. The strain energy functions, which are explicit expressions of systemic stability parameters, of the Neo-Hookean model, the Mooney-Rivlin model and the Ogden model, are derived. Simultaneously, the predigested expression of Ogden model is evaluated. And the deduced results agree with the experimental results. There is great instructional significance in actuator design and fabrication.

1 Zhao X H, Suo Z G. Method to analyze electromechanical stability of dielectric elastomers. Appl Phys Lett, 2007, 91: 061921

2 Zhao X H, Hong W, Suo Z G. Electromechanical coexistent states and hysteresis in dielectric elastomers. Phys Rev B, 2007, 76: 134113

3 Suo Z G, Zhao X H, Greene W H. A nonlinear field theory of deformable dielectrics. J Mech Phys Solids, 2008, 56: 476–486

4 Zhou J X, Hong W, Zhao X H, et al. Propagation of instability in dielectric elastomers. I. J Solids Struct, 2008, 45: 3739–3750

5 Zhao X H, Suo Z G. Electrostriction in elastic dielectrics undergoing large deformation. J Appl Phys, 2008, 104: 123530

6 Liu Y, Liu L, Zhang Z, et al. Comment on "Method to analyze electromechanical stability of dielectric elastomers". Appl Phys Lett, 2008, 93: 106101

7 Norrisa A N. Comment on "Method to analyze electromechanical stability of dielectric elastomers". Appl Phys Lett, 2007, 92: 026101

8 Díaz-Calleja R, Riande E, Sanchis M J. On electromechanical stability of dielectric elastomers. Appl Phys Lett, 2008, 93: 101902

9 Liu Y J, Liu L W, Sun S, et al. Comment on "On electromechanical stability of dielectric elastomers". Appl Phys Lett, 2009, 94: 096101

10 Leng J S, Liu L W, Liu Y J, et al. Electromechanical stability of dielectric elastomer. Appl Phys Lett, 2009, 94: 211901

11 Liu Y J, Liu L W, Sun S H, et al. Stability analysis of dielectric elastomer film actuator. Sci China Ser E-Tech Sci, 2009, 52: 2715–2723

12 Liu Y J, Liu L W, Zhang Z, et al. Dielectric elastomer film actuator: Characterization, experiments and analysis. Smart Mater Struct, 2009, 18: 095024

13 Liu Y J, Liu L W, Yu K, et al. An investigation on electromechanical stability of dielectric elastomer undergoing large deformation. Smart Mater Struct, 2009, 18: 095040

14 Liu Y J, Liu L W, Leng J S, et al. Erratum: "Electromechanical stability of dielectric elastomer" [Appl Phys Lett, 94, 211901 2009]. Appl Phys Lett, 2009, 95: 019902

15 Liu Y J, Liu L W, Zhang Z, et al. Electromechanical stability of electro-active silicone filled with high permittivity particles undergoing large deformation. Smart Mater Struct, 2010, 19: 115025

16 Liu L J, Fan J M, Zhang Z, et al. Analysis of the novel strain responsive actuators of silicone dielectric elastomer. Adv Mater Res, 2008, 47-50: 298–301

17 Liu Y J, Shi L, Liu L W, et al. Inflated dielectric elastomer actuator for eyeball's movements: fabrication, analysis and experiments. SPIE, 2008, 6927: 69271A

18 Liu L W, Liu Y J, Zhang Z, et al. Stability analysis of dielectric elastomer using the elastic strain energy functions with two material con-

stants. SPIE, 2009, 7287: 728719

19 Gyure M F, Beale P D. Dielectric breakdown in continuous models of metal-loaded dielectrics. Phys Rev B, 1992, 46: 3736

20 Xu T B, Su J. Theoretical modeling of electroactive polymer-ceramic hybrid actuation systems. J Appl Phys 2005, 97: 034908

21 Plante J S, Dubowsky S. Large-scale failure modes of dielectric elastomer actuators. Int J Solids Struct. 2006, 43: 7727–7751

22 Leng J S, Zhang Z, Liu L W, et al. Thermodynamics and thermo-electro-mechanical stability of dielectric elastomers composite (in Chinese). Sci China Phys Mech Astron, in Press, doi: 10.1360/132011-480

23 Liu L W, Liu Y J, Li B, et al. Thermo-electro-mechanical stability of dielectric elastomers. Smart Mater Struct, 2011, 20: 075004

24 Mockensturm E M, Goulbourne N. Dynamic response of dielectric elastomers. Int J Non-Linear Mech, 2006, 41: 388–395

25 Goulbourne N C, Mockensturm E M, Frecker M I, et al. Electro-elastomers: Large deformation analysis of silicone membranes. Int J Solids Struct, 2007, 44: 2609–2626

26 Arruda E M, Boyce M C. On the dynamic electromechanical loading of dielectric elastomer membranes. J Mech Phys Solids, 1993, 41: 389–412

27 Corona E, Kyriakides S. On the collapse of inelastic tubes under combined bending and pressure. Int J Solids Struct. 1998, 24: 505–535

28 Kofoda G, Wirges W, Paajanen M, et al. Energy minimization for self-organized structure formation and actuation. Appl Phys Lett, 2007, 90: 081916

29 Kofod G, Sommer-Larsen P, Kronbluh R, et al. Actuation response of polyacrylate dielectric elastomers. J Intell Mater Syst Struct, 2000, 14: 787–793

30 Yoseph B. Electroactive Polymer (EAP) Actuators as Artificial Muscle: Reality, Potential and Challenges. SPIE Press Monograph, 2001. 482

31 Pelrine R, Kornbluh R, Pei Q B, et al. High-speed electrically actuated elastomers with strain greater than 100%. Science, 2000, 287: 836–839

32 Liu L W, Sun S H, Liu Y J, et al. Constitutive relation electromechanical stability analysis of ogden type dielectric elastomerwithlinear permittivity. Chin J Solid Mech, 2010, 31: 181–192

33 Liu Y J, Liu L W, Sun S H, et al. Electromechanical stability of Mooney-Rivlin-type dielectric elastomer with nonlinear variable dielectric constant. Polym Int, 2010, 59: 371–377

34 Liu Y J, Liu L W, Zhang Z, et al. Analysis and manufacture of an energy harvester based on a Mooney-Rivlin–type dielectric elastomer. Europhys Lett, 2010, 90: 36004

35 Li B, Liu L W, Suo Z G. Extension limit, polarization saturation, and snap-through instability of dielectric elastomers. Int J Smart Nano Mater, 2011, 2: 59–67

36 Kong X H, Li Y B, Liu L W, et al. Electromechanical stability of semi-crystalline polymer. Thin Solid Films, 2011, 519: 5017–5021

37 Carpi F, Rossi D De. Improvement of electromechanical actuating performances of a silicone dielectric elastomer by dispersion of tita-nium dioxide powder. IEEE Trans Dielectr Electr Insul, 2005, 12: 835–843

38 Zhang Z, Liu L W, Fan J, et al. New silicone dielectric elastomers with a high dielectric constant. SPIE, 2008, 6926: 692610