In-plane elasticity of a novel auxetic honeycomb design

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ABSTRACT

This work presents a novel negative Poisson’s ratio honeycomb design composed by two parts (a re-entrant hexagonal component and a thin plate part) that provide separate contributions to the in-plane and out-of-plane mechanical properties. The re-entrant hexagons provide the in-plane negative Poisson’s ratio, the in-plane compliance and the out-of-plane compressive strength, while the thin plate part connecting the re-entrant hexagonal section bears the large out-of-plane flexibility. This paper focuses on the in-plane mechanical properties of the auxetic cellular structure. Theoretical models related to the in-plane uniaxial tensile modulus, the shear modulus, and the Poisson’s ratios have been built and validated using the finite element techniques. The in-plane behavior of the honeycomb has also been investigated against the geometrical parameters of the unit cell using a parametrical analysis. The theoretical and numerical models illustrate good agreement and show the potential of its application in morphing structures. We also provide a benchmark of the auxetic configuration proposed in this work against negative Poisson’s ratio topologies from open literature.

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1. Introduction

Due to the significant lightweight and designable in-plane and out-of-plane mechanical performances honeycomb structures have attracted worldwide attention for several decades [1,2]. The primary application of honeycomb structures is to be used as sandwich core materials in various engineering fields, such as aerospace, marine and automotive lightweight structures [3,4]. In recent years, honeycombs have also been proposed as a promising solution for the morphing skin [5–8]. Different honeycomb configurations result in different values of the in-plane Poisson’s ratio. A variety of honeycomb configurations achieving positive/negative/zero Poisson’s ratio have been proposed and investigated with special focus on the in-plane mechanics, out-of-plane compressive strength and the transverse shear modulus [9–28]. Honeycombs with negative Poisson’s ratio (NPR) can likewise be described as auxetic [28–27]. When subject to out-of-plane bending deformation honeycombs with positive Poisson’s ratio (PPR) show anti-elastic or saddle-shape curvature that somehow limits their application in sandwich structures with complex out-of-plane geometry [39,30]. No lateral deformations can be observed when honeycombs exhibiting zero Poisson’s ratio are loaded along one in-plane direction making them more suitable for one-dimensional (span) morphing [6,31]. NPR honeycomb structures feature synclastic curvature behavior, therefore making possible to provide dome-shaped surfaces when loaded with out-of-plane bending deformation [30,32]. Auxetic honeycombs have been employed to prototype radomes [33], adaptive and deployable structures [34], and morphing wings [35–37].

A ZPR cellular configuration made by combinations of hexagons and thin plates has been previously proposed by the authors to achieve large out-of-plane deformation abilities and separate tailorable design of the in-plane and out-of-plane performances [38,39]. In-plane mechanics and the bending property of the ZPR cellular structure have been investigated. A comparison of the out-of-plane bending behavior of different types of cellular configurations with the same relative density using the 3-point bending tests show that inserting the thin plate can achieve an enhanced bending compliance [39]. In this work, a similar honeycomb design exhibiting in-plane negative Poisson’s ratio for large out-of-plane deformations and morphing applications is proposed and investigated. A re-entrant hexagonal structure and a thin plate are consisted in the honeycomb configuration. The re-entrant hexagonal...
section is designed to bear the out-of-plane compression and to produce in-plane negative Poisson’s ratio and flexibility, while the thin plate is inserted for the large out-of-plane compliance. As a result, different sections bring about different mechanical properties leading to a separate design for the in-plane and out-of-plane performances. Analytical models to describe the in-plane elasticity of the novel honeycomb have been developed and validated using the Finite Element (FE) homogenization approaches. The dependence of the in-plane properties on the honeycomb geometrical parameters has also been further investigated using a combination of analytical models and FE homogenizations.

2. Geometry of the novel auxetic honeycomb design

The layout of the novel negative Poisson’s ratio honeycomb structures and geometry of its unit cell are presented in Fig. 1. It’s evident that the unit cell possesses a vertical and horizontal symmetry. The unit cell can be described by a re-entrant hexagonal structure and two thin plates connecting the re-entrant hexagonal sections. The lengths of the inclined wall and two vertical walls are represented by parameters \( l, h = \alpha l \) and \( h_1 = \mu l \), respectively. The internal cell angle is represented by the parameter \( \theta \). The parameter \( t = \beta l \) represents the thickness of the re-entrant hexagonal section. The dimensions of the thin plate are demonstrated by the length \( l_1 = \eta l \), the width \( h = \alpha l \) and the thickness \( b_1 = \gamma l \). The thickness of the whole unit cell is represented by the parameter \( b = \gamma l \). It should be noted that, to avoid the overlapping of cell walls the geometric constraints \( \mu > \sin \theta \)
\( \alpha > (\beta(2 + \sin \theta) + 2 \sin \theta \cos \theta)/\cos \theta \) must be satisfied. In addition, the relative density of the novel auxetic honeycomb can be calculated by inspection as:

\[
\frac{\rho}{\rho_c} = \frac{2a\eta l^2 + \gamma l^2 [a(2 + 2 \cos \theta) - 2 \sin \theta \cos \theta + 2\beta(\mu - \sin \theta)] - 2c_1 \gamma \cos \theta (a_1 - c_1 \sin \theta)]}{\gamma l^2(2\eta + 2 \cos \theta + \beta)(a + 2\mu - 2 \sin \theta)}
\]

\((1)\)

In \((1)\) \( a_1 = a \alpha - 2\beta l \cot \left(\frac{\pi}{3} - \frac{\theta}{2}\right) \) and \( c_1 = l - \beta l/(2 \cos \theta) \) are respectively the length of the inner side of the vertical and the inclined walls of the unit cell.

3. Theoretical models

The analytical models to calculate the in-plane tensile modulus of the novel auxetic honeycomb structure developed in this paper are based on the application of the Castiglano’s second theorem [40]. The honeycomb ribs are assumed to undergo bending and axial tensile deformation. The elastic modulus along the 1 (horizontal) direction of the honeycomb is calculated following the loading scheme shown in Fig. 2 (a). The inclined walls are bent and stretched by the applied uniformly distributed stress \( \sigma_1 \) which is parallel to the 1 direction. It is evident that \( F_1 = 0 \), \( F_1 = \sigma_1 b(h - 2lsin\theta + 2 \mu l)/2 \) and the bending moment \( M_1 \) can be obtained from the equilibrium equations [1]:

\[
M_1 = \frac{1}{2}F_1 l \sin \theta
\]

\((2)\)

In this work, the positive bending moment is orientated along the anti-clockwise direction. The bending moment distribution and the axial tensile force on the single rib are:

\[
M_1(x) = \left(\frac{1}{2}l - x\right)F_1 \sin \theta, F_{N1}(x) = F_1 \cos \theta
\]

\((3)\)

According to the Castiglano’s second theorem, one obtains the strain energy of the single bending and stretched wall:

\[
U_1 = \frac{F_1^2 l^3 \sin^2 \theta}{24E_1 l} + \frac{F_1^2 l \cos^2 \theta}{2E_1 A}
\]

\((4)\)

In \((4)\), \( E_1 \) is the Young’s modulus of the honeycomb material, \( l = \beta\gamma l^2/12 \) and \( \beta = \gamma l^2 \) the second moment of area and the area of the cross section of the inclined wall respectively. Using the Castiglano’s second theorem herein, it is possible to calculate the horizontal deformation of the inclined wall:

\[
\delta_{11} = \frac{F_1 l^3 \sin^2 \theta}{12E_1 l} + \frac{F_1 l \cos^2 \theta}{E_1 A}
\]

\((5)\)

In order to take the thin plate into account, the axial tensile deformation induced by the applied stress \( \sigma_1 \) can be calculated as:

\[
\delta_{12} = \frac{2F_1 \eta}{E_1 a \mu l}
\]

\((6)\)

Fig. 1. Geometry of the novel auxetic honeycomb structures.
The total deformation of the unit cell along the 1 (horizontal) direction can be therefore obtained as:

$$d_1 = 2(d_{11} + d_{12}) \quad (7)$$

With equation (7) and the geometry of the unit cell shown in Fig. 1, one can obtain the homogenized strain along the horizontal direction:

$$\varepsilon_1 = d_1 / 2(\eta l + l \cos \theta) + \beta l \quad (8)$$

The homogenized and non-dimensional Young's modulus along the 1 direction can therefore be obtained from the ratio between the tensile stress \( \sigma_1 \) and strain \( \varepsilon_1 \):

$$E_1 \equiv E_s \frac{\alpha \lambda \beta^3 \left( \eta + \cos \theta + \frac{\beta}{2} \right)}{\left( \frac{1}{2} \alpha + \mu - \sin \theta \right) \left[ \alpha l \left( \sin^2 \theta + \beta^2 \cos^2 \theta \right) + 2\eta \gamma \beta^3 \right]} \quad (9)$$

Following the same procedure, one can also obtain the homogenized and non-dimensional Young's modulus along the 2 direction. The loading scheme to calculate \( E_2/E_s \) is shown in Fig. 2(b). A uniformly distributed stress \( \sigma_2 \) has been applied on the unit cell along the 2 (vertical direction). It is also evident that \( P_2 = 0, F_2 = \sigma_2 \gamma l(2\eta l + 2\cos \theta + \beta)/2 \) and from the equilibrium equations the bending moment can be calculated as:

$$M_2 = \frac{1}{2} F_2 l \sin \theta \quad (10)$$

The bending moment and the tensile force distribution along the inclined wall are given by:

$$M_2(x) = \left( \frac{1}{2} l - x \right) F_2 \cos \theta, F_{N2}(x) = F_2 \sin \theta \quad (11)$$

Again following the Castigliano’s second theorem, one can obtain the strain energy and deformation along the 2 (vertical) direction of the bent and stretched inclined wall:

$$U_2 = \frac{F_2^2 \beta \cos^2 \theta + F_2^2 l \sin^2 \theta}{24E_s l} + \frac{F_2^2 l \sin^2 \theta}{2E_s A} \quad (12)$$

$$\delta_{21} = \frac{F_2 \lambda \cos \theta + \frac{F_2 l \sin^2 \theta}{E_s A}}{12E_s l} \quad (13)$$

In order to determine the dependence of the tensile modulus along the 2 direction on the geometric parameters of the central vertical wall, the axial tensile deformation of the central vertical wall has been taken into account herein:

$$\delta_{22} = \frac{2F_2 \mu}{E_s \beta \gamma l} \quad (14)$$

Thus, one obtains the total deformation and strain of the unit cell induced by the applied uniformly distributed stress along the 2 direction as shown in Fig. 2(b),

$$\delta_2 = 2(\delta_{21} + \delta_{22}) \quad (15)$$

$$\varepsilon_2 = \frac{\delta_2}{d_2 + 2\mu - 2l \sin \theta} \quad (16)$$

The homogenized nondimensional Young's modulus along the 2 direction can also therefore be obtained from the ratio between the tensile stress \( \sigma_2 \) and strain \( \varepsilon_2 \):

$$E_2 \equiv E_s \frac{\beta^3 \left( \frac{\alpha}{2} + \mu - \sin \theta \right)}{\left( \eta + \cos \theta + \frac{\beta}{2} \right) \left( 2\mu \beta^2 + \beta^2 + \cos^2 \theta - \beta^2 \cos^2 \theta \right)} \quad (17)$$

The Poisson's ratio of the novel auxetic honeycomb can be calculated by the definition of taking the negative ratio of the strain normal to over the strain parallel to the loading direction. When loading in the 1 (horizontal) direction, one can obtain the Poisson ratio as:
According to Table 8.12 in reference [40], one T-beam must be found. The geometry of the cross section of the T-beam is shown in Fig. 3. A uniformly distributed shear stress has been applied on the unit cell, and because of the vertical symmetry a half model of the unit cell is used in the following calculation as shown in Fig. 2(c). Due to the horizontal symmetry of the half model, there is no relative displacement among the points B, C, D, leading to that the shear deformation of the half model simplified into the sum of the horizontal deformations of the honeycomb struts AC and BE induced by applying the shear stress. In this work, the bending deformation of AC and BE, as well as the shear deformation of BE have been taken into account. It’s evident that the relationship between the force $F_1$ and the applied stress $\tau$ can be described as:

$$F_1 = \tau \gamma (2\eta l + 2l \cos \theta + \beta l) \quad \text{(20)}$$

The horizontal displacement of the point A against the point C can be obtain using the standard equation of the Euler-Bernoulli beam theory [40].

$$\delta_{121} = \frac{\tau \gamma \mu^3 F_1 (2\eta + 2 \cos \theta + \beta)}{3E l} \quad \text{(21)}$$

The inclined wall and the central vertical wall have a same cross section, so the second moment of area of the cross section of the wall AC is also marked as $I$. In order to calculate the horizontal deformation of the honeycomb strut BE, the flexural center of this T-beam must be found. The geometry of the cross section of the T-beam is shown in Fig. 3. According to Table 8.12 in reference [40], one can find the distance between the flexural center and the origin of coordinates.

$$e = \frac{1}{2} (\beta l + \eta l) \frac{(\beta l)^3 \eta l}{(\beta l)^3 \eta l + (\gamma l)^3 \beta l} \quad \text{(22)}$$

For a thin-walled I-beam the central wall carries the great majority of the shear stresses, and that is also appropriate for a thin-walled T-beam. In this paper only the shear deformation of the central wall (the red dashed rectangle in Fig. 3) of the T-beam has been taken into account. According to the Timoshenko beam theory, the horizontal deformation of the strut BE which is a cantilever beam with a concentrated force loading at the free end can be calculated by:

$$\delta_{122} = \frac{F_1 (a \ell)^3}{48E I_1} + \frac{F_1 a \ell}{4K A_1 G_s} \quad \text{(23)}$$

In equation (23), $G_s = E_s (2 + 2\nu_s)$ is the shear modulus of the honeycomb material, $\nu_s$ the Poisson’s ratio of the honeycomb material, $K$ the Timoshenko shear coefficient of the red dashed rectangle in Fig. 3, $I_1$ and $A_1$, the second moment of area and the area of the cross section of the T-beam respectively. Evidently, $A_1 = (\eta + \beta) \beta l^2$ and the coefficient $K$ [41] and the second moment of area $I_1$ can be calculated following:

$$K = \frac{10 (1 + \nu_s)}{12(1 - \nu_s)} \frac{1}{I_1} \quad \text{(24)}$$

$$I_1 = \frac{\gamma l^4}{24} \left( \frac{\lambda^3 \eta (\eta + \beta)}{\lambda^3 \eta + \gamma^3 \beta} + \beta \right)^3 - \left( \frac{\lambda^3 \eta (\eta + \beta)}{\lambda^3 \eta + \gamma^3 \beta} - \beta \right)^3 \quad \text{(25)}$$

The shear strain of the half model of the unit cell is given by:

$$\gamma_{12} = \frac{\delta_{121} + \delta_{122}}{\frac{2}{\ell} + l \sin \theta} \quad \text{(26)}$$

Using $G = \tau / \gamma$, one can obtain the homogenized and nondimensional shear modulus of the novel auxetic honeycomb structure as:

$$G_{12} = \frac{\eta + \mu - \sin \theta}{\gamma (2\eta l + 2l \cos \theta + \beta)} \left( \frac{a \ell}{2l} + \frac{(a \ell)^3}{48l^3} + \frac{a l (1 + \nu_s)}{2K A_1 G_s} \right) \quad \text{(27)}$$

4. Finite element homogenization

In order to validate the analytical models of the in-plane elasticity, the commercial finite element software ANSYS (version 13.0, ANSYS Inc.) was used to perform the numerical homogenization of the novel auxetic honeycomb structures. The models were developed using 3D structural elements (Solid 45) defined by 8 nodes and three translational degrees at each node. The full-size representative volumes were given by $5 \times 5 \times 5$ unit cells (Fig. 4) with boundary conditions following [38,42,43]. According to the convergence tests, a minimum element size of $t/2$ was used in the simulations. The boundary conditions used to get the in-plane constants of the novel auxetic honeycomb were listed in Table 1. To obtain the tensile Young’s modulus $E_s$ and the Poisson’s ratio $\nu_{12}$ the in-plane tension along the 1 direction was simulated with boundary conditions that the surfaces A and B were loaded with displacement conditions $u_1 = 0$, $u_2 = 0$, $u_3 = 0$ where $\delta_0$ was the tensile axial strain and $x_1$ the coordinate of the element nodes on the 1 direction, the surfaces C, D, E and F were set as free boundary conditions.
conditions. The in-plane tension along the 2 direction was also simulated to get the tensile Young’s modulus \( E_2 \) using similar boundary conditions with the simulated tension on the 1 direction as shown in Table 1. Displacements of \( u_2 = e_0 x_2, u_1 = u_3 = 0 \) were loaded on the surfaces C and D, where \( e_0 \) was also the tensile axial strain and \( x_2 \) the coordinate of the element nodes on the 2 direction. Free boundary conditions were also applied on the surfaces A, B, E and F. In the case of the in-plane shear simulation, following [44] a cantilever beam clamp was used to apply the boundary conditions. Surface C was loaded with displacements equal to \( u_2 = g_0 L_2 \), \( u_1 = u_3 = 0 \) where \( g_0 \) was the shear strain, and \( L_2 \) the total length of the representative volumes along the 2 direction, and surface D was set as a fixed end. Mimicking a continuous honeycomb along the 1-direction surfaces A and B were subjected to anti-symmetrical boundary conditions. On surfaces E and F free boundary conditions were also applied. The average tensile stress and strain were calculated within the central unit cell (red ellipse in Fig. 4) [44] to avoid the Saint-Venant effect from the borders. The following equations were employed to calculate the average tensile and shear stresses [10]:

\[
\overline{\sigma_{ii}} = \frac{1}{V} \int \sigma_{ii}(1, 2, 3)dV, \overline{\tau_{12}} = \frac{1}{V} \int \tau_{12}dV \tag{28}
\]

The relations of stress and strain for a general orthogonal anisotropic material can be expressed as:

\[
\begin{bmatrix}
\tau_1 \\
\tau_2 \\
\tau_3 \\
\tau_{13} \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 \\
S_{21} & S_{22} & S_{23} & 0 & 0 \\
S_{31} & S_{32} & S_{33} & 0 & 0 \\
0 & 0 & 0 & S_{55} & 0 \\
0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\gamma_{13} \\
\gamma_{12}
\end{bmatrix} \tag{29}
\]

Thus the in-plane compliance of the novel auxetic honeycomb were given by:

\[
E_1 = \frac{1}{S_{11}}, E_2 = \frac{1}{S_{22}}, \nu_{12} = -\frac{S_{21}}{S_{11}}, G_{12} = \frac{1}{S_{66}} \tag{30}
\]

5. Results and discussions

5.1. The in-plane tensile modulus

For all the calculations in this paper the length of the inclined wall has been set as 10 mm and isotropic properties of \( E_s = 1.6 \) GPa, \( \nu_s = 0.3 \) have been used for the honeycomb material. The existing of
the re-entrant hexagonal section in the novel auxetic honeycomb leads to a similar in-plane tensile stiffness with the one described by the Gibson and Ashby model for the classical centersymmetric hexagonal configuration [1]. Figs. 5 and 6 demonstrate the FE homogenization and analytical predictions of the non-dimensional in-plane elastic modulus $E_1/E_s$ versus the cell angles for various parameters $\alpha$ and $\beta$. In general, the non-dimensional elastic modulus $E_1/E_s$ decreases with increasing cell angles $\theta$ while the other geometrical parameters keep constant. As shown in both figures, the results of the FE simulations and the analytical models show generally good agreements with each other when the cell angle is larger than $10^\circ$, while large discrepancies occur if $\theta \leq 10^\circ$. For example, as shown in Fig. 5 the analytical results is averagely 51.17% stiffer than the FE homogenizations when $\theta \leq 10^\circ$ for $\alpha = 2.0$, but that data for the case of $\theta > 10^\circ$ reduces to 8.64%. Apparently, neglecting the shear deformation in the analytical model results in a slightly stiffer tensile modulus of the novel auxetic honeycomb than the analogous one from the FE simulation when $\theta > 10^\circ$. And when $\theta \leq 10^\circ$ the shear deformation contributes substantially to the whole honeycomb deformation that leads to a relatively larger discrepancy between the analytical and FE results. Increasing of the parameter $\alpha$ will decrease the nondimensional tensile modulus of the novel auxetic honeycomb along the 1-direction while other geometrical parameters keep constant. On the opposite, the nondimensional tensile modulus on the 1-direction increases substantially while the parameter $\beta$ increases, for instance the in-plane tensile modulus shows an averagely increase of more than 2600% when the parameter $\beta$ varies from 0.05 to 0.15. It provides a good way to design the in-plane tensile modulus by varying parameter $\beta$. Figs. 7 and 8 show the values of the nondimensional modulus $E_2/E_s$ from FE homogenizations versus the cell angles at different $\gamma$ and $\mu$ values. Decreases are observed on these two figures when the values of $\gamma$ and $\mu$ increase keeping the rest parameters constant.

Figs. 9–12 demonstrate the variation of the non-dimensional in-plane tensile modulus on the 2-direction ($E_2/E_s$) versus cell angles...
for different geometrical parameters of the unit cell. The analytical equation shows a very close comparison with the trends followed by the results of the FE homogenization. Neglecting the deformation of the T-beam and the shear deformation of the inclined wall results in a slightly stiffer analytical tensile modulus than the datas obtained through the FE homogenizations. For example, the analytical results is averagely 10.30% stiffer than the results from FE simulations when the parameter $\alpha$ equal to 2.0. Generally, the non-dimensional tensile modulus decreases slightly and then increase clearly when the cell angle varies from $2^\circ$ to $40^\circ$. From Fig. 9 one can draw the conclusion that increasing of the parameter $\alpha$ leads to an increase of the in-plane tensile modulus. Fig. 10 shows a significant increase of the non-dimensional tensile modulus when the parameter $\beta$ increases. An increase of the more than 1200% of the non-dimensional tensile modulus show up when the parameter $\beta$ varies from 0.05 to 0.125 which makes varying $\beta$ also a good design method for $E_2/E_s$. Figs. 11 and 12 illustrate the analytical predictions of the non-dimensional in-plane elastic modulus $E_2/E_s$ versus the cell angles for various parameters $\eta$ and $\mu$. An increase of the parameter $\eta$ leads to a decrease of the non-dimensional tensile modulus. Conversely, an increase of the parameter $\mu$ results in an increase of the same mechanical property.

5.2. Poisson’s ratio

Figs. 13–16 show the variation of the in-plane Poisson’s ratio $\mu$ of the novel auxetic honeycomb versus cell angles for different geometrical parameters. It must be pointed out that the existing of the re-entrant hexagonal section in the novel auxetic honeycomb structure produces a similar in-plane Poisson’s ratio that also predicts infinite values in the vicinity of an internal cell angle of $0^\circ$ as the one described by Gibson and Assby [1]. But when the cell angle is relatively large (larger than $15^\circ$) the analytical results show good agreements with the analogous datas from the FE homogenization. To make it clear, the analytical Poisson’s ratio is only 3.80% over the one predicted by the FE homogenization when $\theta = 30^\circ$ and $\alpha = 1.4$, $\beta = 0.1$, $\eta = 0.1$, $\gamma = 1.0$, $\lambda = 0.1$, $\mu = 1.0$. The FE homogenization of the in-plane Poisson’s ratio versus cell angles for different geometrical parameters have been illustrated in Figs. 13–16. Apparently, the Poisson’s ratio reduces quickly to 0 when the cell angle is less than a certain degree ($10^\circ$ in Fig. 13), increases slowly when the cell angle is larger than the certain degree leading to a smallest value of the Poisson ratio at that value of the cell angle. The increasing of the cell wall aspect ratio $\alpha$ leads to an increasing of the Poisson’s ratio. And the Poisson’s ratio also increases when the parameter $\beta$ increases. Moreover, the increasing of the parameter $\beta$ would also lead to an increase of the certain value that dividing the trends of the Poisson’s ratio against the cell angles. The certain degree of the cell angle increases from $8^\circ$ to $14^\circ$ when the parameter varies from 0.05 to 0.15. Fig. 15 show the FE homogenization of the Poisson’s ratio $\mu$ versus the cell angles for various parameters $\eta$ while $\alpha = 2.0$, $\beta = 0.1$, $\gamma = 1.0$, $\lambda = 0.1$, $\mu = 1.0$. An increase of the parameter $\eta$ leads to a slightly increase of the Poisson’s ratio when the cell angle is less than the certain degree, but the Poisson’s ratio shows an opposite trend when the cell angle is larger than the certain degree. Fig. 16 shows the values of the
Poisson’s ratio from FE homogenizations versus cell angles at different \( \mu \) values and constant \( a = 2.0, \beta = 0.1, \gamma = 1.0, \lambda = 0.1, \mu = 1.0 \). The Poisson’s ratio show clearly increases when the geometrical parameter \( \mu \) varies from 0.5 to 1.0.

5.3. In-plane shear modulus

Generally, there are three major boundary conditions (uniaxial, biaxial and tangential loading \([45]\)) can be used to simulate a pure in-plane shear deformation. In this work, the tangential loading has been used in the FE homogenizations, and the analytical equation of the in-plane shear modulus is obtained using a pure shear model. Even the fixture of the tangential loading provide a pure shear deformation as closely as the ideal one, there are still some differences between a real pure in-plane shear deformation and an approximate one \([44]\). In the procedure of the calculation of the in-plane shear modulus, assuming that the web of the thin-walled T-beam bears all the shear stress and neglecting the shear stress in the flanges of the T-beam also produces some difference between the analytical and FE results. However, the analytical results show good agreements with the analogous ones from the FE
homogenizations. A maximum discrepancy of only 2.60% has been observed between the two results when the cell angle varies from $0^\circ$ to $38^\circ$ for $\alpha = 1.6$, $\beta = 0.1$, $\eta = 1.0$, $\lambda = 0.1$, $\mu = 1.0$. Figs. 17 and 18 show the variation of the non-dimensional in-plane shear modulus against the internal cell angles for different geometrical parameters $\alpha$ and $\beta$ of the unit cell. The non-simensional in-plane shear modulus decreases when the cell angle increases. On the contrary, an increase of cell wall aspect ratios $\alpha$ and $\beta$ leads to also an increase of the non-dimensional in-plane shear modulus. It must be noted that even a small increase of the parameter $\beta$ can lead to a large increase of the non-dimensional in-plane elastic modulus providing that varying $\beta$ is a good way for the design of the in-plane shear modulus. From Fig. 18, the discrepancy between the analytical and FE results also increases when the parameter $\beta$ increases. That's because the shear deformation of the central vertical wall has been neglected in the analytical model and taken into account in the FE homogenizations. Increasing of the parameter $\beta$ leads to an increase of the ratio of the shear deformation of the central vertical wall over the whole deformation of the unit cell. Figs. 19–21 demonstrate the analytical predictions of the non-dimensional in-plane elastic modulus $G_{12}/E_s$ versus the cell angles for various parameters $\gamma$, $\lambda$, $\mu$ respectively while the other parameters keep constant. Evidently, an increase of the three parameters all lead to the increasing of the non-dimensional in-plane elastic modulus.

5.4. Comparison with existing auxetic configurations

A benchmark of the in-plane mechanics of the cellular configuration against three different auxetic cellular structures is shown in Fig. 22. The results for the re-entrant hexagonal configurations have been calculated using the typical formulas contained in Ref. [1], for internal cell angles ranging from $-40^\circ$ to $-10^\circ$ and cell wall aspect ratios equal to $1.0$, $1.2$, $1.4$, $1.6$, $1.8$, $2.0$. The choice of these angles stems from avoiding the extremely large anisotropy along the 1-direction when the internal cell angle is near to zero. The data representing the anti-tetra-chiral are obtained using equations sourced from Ref. [46], for cell wall aspect ratio $r/L$
varying from 0.05 to 0.45 and a constant cell wall thickness ratio $t/L$ of 0.1. The results of the novel auxetic configuration have also been calculated with internal cell angles ranging from $-40^\circ$ to $-10^\circ$, and cell wall aspect ratios of 1.0, 1.2, 1.4, 1.6, 1.8, 2.0 while the other parameters keep constants. The novel auxetic honeycomb presented in this work shows very similar results to the classical re-entrant hexagonal configuration for the tensile modulus along the 1-direction, the Poisson’s ratio and the in-plane shear modulus. The tensile modulus along the 2-direction of the re-entrant configuration however varies between 3.27 and 17.20, while the novel auxetic structure tends to cluster in a relative small interval (7.20–12.51). The same can be also stated for the in-plane Poisson’s ratios and modulus along the 1-direction, with most of the results belonging to the new configuration being clustered in a non-dimensional density interval $\rho/\rho_c/\beta$ between 1.05 and 1.7. Note that the dimensionless Young’s modulus along the 2-direction of the anti-tetrachiral honeycomb is equal to the one along the 1-direction because of isotropy. The insertion of thin plate parts in the novel configuration appears to provide a smaller relative density when compared with the one of the re-entrant hexagonal honeycomb. The anti-tetra-chiral honeycomb generally provides a much higher relative density when compared with the other two cellular configurations. The in-plane Poisson’s ratio of the anti-tetra-chiral honeycomb is also very close to $-1$, even when the

Fig. 21. Analytical predictions of the non-dimensional in-plane elastic modulus $G_{12}/E_s$ versus the cell angles for various parameters $\mu$ while $\alpha = 2.0$, $\beta = 0.1$, $\gamma = 1.0$, $\lambda = 0.1$.

Fig. 22. Comparison of the in-plane elasticity between the novel auxetic honeycomb, the classical re-entrant configuration and the anti-tetra-chiral topologies.
relative density increases. One of the biggest features of the new cellular configuration is the high specific shear stiffness at very low dimensionless densities (below 1.4), as well as the negative in-plane Poisson’s ratio higher (in terms of magnitude) than the one provided by the classical re-entrant of butterfly shape at $\rho \alpha / \beta$ lower than 1.25.

6. Conclusions

In this work a novel auxetic honeycomb structure for large out-of-plane deformations and morphing structures has been proposed and the novelty of the auxetic honeycomb topology assembles two geometrical entities together: the re-entrant hexagonal structures to provide the out-of-plane compressive strength and to produce in-plane negative Poisson’s ratio and compliance, and a thin plate that inserted for the large out-of-plane flexibility. The two entities bring about different contributions to the homogenized mechanical properties leading to a separate design for the in-plane and out-of-plane performances. The analytical models for the in-plane tension modulus along both the two directions, the in-plane Poisson’s ratio and the in-plane shear modulus have been developed and validated by FE homogenizations. The parametric analyses show that it is possible to obtain large variations and control of the design of the in-plane mechanics through the variation of the unit cell geometric parameters making this novel auxetic honeycomb structure particularly interesting for morphing structures.

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