

具有线性介电常数的 Ogden 型介电弹性体的本构关系和机电稳定性*

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摘要 应用多材料常数的 Ogden 弹性应变能函数分析了介电弹性体的力学行为, 研究了介电弹性体的机电稳定性. 数值结果表明, 通过对材料系数(如材料常数比和电致伸缩系数等)的恰当调节可以使得介电弹性体材料或介电弹性体结构更趋稳定. 这些有益于深入理解介电弹性体的机电稳定性行为, 进而设计恰当的介电弹性体器件.

关键词 机电稳定性, 介电弹性体, Ogden 弹性应变能函数, 线性介电常数

0 引言

电致活性聚合物 (Electroactive polymer, EAP) 是一类能够在外加电场诱导下, 通过改变内部构造产生多种形式力学响应的材料, 具有特殊的机械性能和电性能. 介电弹性体 (Dielectric elastomers, DE) 是制造智能主动驱动器最有潜力的电致活性聚合物材料, 也称电致活性聚合物人工肌肉 (Electroactive polymer muscle, EPAM). 它在外加电场下能够产生较大的应变及作用力, 具有超大变形 (380%)、高弹性能密度 (3.4 J/g)、超短反应时间、高疲劳寿命、易于加工、易于成形、高效率、轻质量、低价格、高循环次数等优良特性^[1-12]. 在介电弹性体两个相对表面均匀涂覆柔性电极, 施加电压后, 由于静电力作用, 介电弹性体薄膜将产生厚度方向变薄而平面方向变大^[13-20]. 厚度的变薄将导致施加在介电弹性体的电场变高, 此正反馈过程一直持续下去, 当电场达到或超过介电弹性体的临界击穿电场时, 薄膜被击穿, 介电弹性体机电耦合系统失稳^[21-25].

机械力场和电场耦合作用在介电弹性体上将引起机电耦合系统电击穿从而使系统失稳^[21-39]. 赵和锁提出介电弹性体非线性机电稳定性分析理论后, 介电弹性体机电耦合系统的失效和非线性机电稳定性分析越来越深入、全面和具体^[28-40]. 赵和锁提出可以应用任意的自由能函数分析介电弹性体机电稳定性, 他们应用一个材料常数的 neo-Hookean 弹性应变能函数分析理想弹性体的机电稳定性行为, 分

别研究对介电弹性体施加等双轴预应力和非等双轴预应力时系统的机电稳定性性质, 描绘出介电弹性体名义电位移和名义电场的关系, 首次从理论的角度证明预拉伸能够增加介电弹性体临界名义电场, 进而显著提高其机电稳定性这一实验现象, 而且计算出的介电弹性体临界击穿电场和实验结果吻合的很好^[28-30]. 刘等应用两个材料常数的弹性应变能函数分析介电弹性体的机电稳定性行为, 引入材料常数比 k 对于分析不同类型和结构的介电弹性体的机电稳定性有巨大帮助, 给出不同介电弹性体的名义电位移和名义电场的关系^[34], 得出当 k 增大时, 介电弹性体的机电稳定性显著增强, 它能够帮助指导介电弹性体驱动器的设计和制造. Norris A N 应用 Ogden 弹性应变能函数模型分析介电弹性体的机电稳定性行为, 精确推导出临界真实电场、名义应力与拉伸率的关系, 并给出应用 Ogden 弹性应变能模型的简化形式时, 临界真实电场、名义应力与预拉伸率的精确关系^[32]. Diaz-Calleja R 等对 neo-Hookean 型硅橡胶的机电稳定性行为进行了深入的研究, 给出介电弹性体硅橡胶在两种特殊载荷情况下的 Hessian 矩阵, 得到形象而直观介电弹性体的稳定区域和不稳定区域帮助更深入理解 neo-Hookean 型硅橡胶的机电稳定性行为^[33]. 在此基础上, 刘等研究了 Mooney-Rivlin 型硅橡胶的机电稳定性区域, 给出硅橡胶机电稳定的临界控制条件, 并计算出两种拉伸率情况下的稳定性区域和不稳定性区域, 为设计和制备这种材料

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提供理论指导^[37].最近,笔者给出应用多个材料常数的弹性应变能函数模型分析理想介电弹性体薄膜驱动器机电稳定性的分析方法^[39].

上述研究应用的电场能量密度函数中的介电常数是不变化的,是一个常数.近期赵和锁的根据 Wissler 等对介电弹性体介电常数的实验,提出把介电常数拟合为依赖拉伸率的线性变化函数^[35],在此基础上,分析经历大变形的介电弹性体的力学行为,同时应用一个材料常数的 neo-Hookean 弹性应变能函数研究介电弹性体的机电稳定性.

本文应用多材料常数的 Ogden 弹性应变能函数耦合线性介电常数的电场能量密度函数构建系统的自由能函数,基于此研究系统的力学性能和机电稳定性性能.计算出两种特殊载荷情况下的名义电场和名义电位移,给出稳定性参数的变化规律,结果表明,材料常数比 k 值越大和电致伸缩系数 r 越小的介电弹性体材料或结构越稳定.

1 基本理论和延伸

锁等在最近的工作中提出,经历大变形的介电弹性体介电常数并不是独立的,它依赖于弹性体本身的变形,这和研究人员得到的实验结论是一致的^[19,36].在本研究中,为分析介电弹性体机电耦合系统的力学行为和机电稳定性行为,应用介电常数变化的电场能量密度函数模型.引入介电常数的线性表达式如下^[35]:

$$(\epsilon_1, \epsilon_2, \epsilon_3) = [1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] \bar{\epsilon} \quad (1)$$

这里 $(\epsilon_1, \epsilon_2, \epsilon_3)$ 是经历大变形后的介电弹性体的介电常数, $\bar{\epsilon}$ 是未经历大变形的介电弹性体的介电常数, a 和 b 是介电弹性体的电致伸缩系数, λ_1, λ_2 和 λ_3 分别是介电弹性体平面方向和厚度方向的拉伸率.

根据介电弹性体的非线性场理论,其机电耦合系统的自由能函数可以写成如下表达^[28-39]:

$$W(\epsilon_1, \epsilon_2, \epsilon_3, D^-) = W(\epsilon_1, \epsilon_2, \epsilon_3) + W(\epsilon_1, \epsilon_2, \epsilon_3, D^-) \quad (2)$$

其中 $W(\epsilon_1, \epsilon_2, \epsilon_3, D^-)$ 为机电耦合系统的自由能函数, $W(\epsilon_1, \epsilon_2, \epsilon_3)$ 为系统的弹性应变能量函数, $(\epsilon_1, \epsilon_2, \epsilon_3, D^-)$ 为系统的电场能量密度函数.

$$W(\epsilon_1, \epsilon_2, \epsilon_3, D^-) = \frac{D^{-2}}{2(\epsilon_1, \epsilon_2, \epsilon_3)} \epsilon_1^{-1} \epsilon_2^{-1} \epsilon_3 \quad (3)$$

整个系统的名义应力 s_1, s_2, s_3 和名义电场 E^- 由下式确定:

$$\begin{cases} s_1 = \frac{\partial W(\epsilon_1, \epsilon_2, \epsilon_3, D^-)}{\partial \lambda_1} \\ s_2 = \frac{\partial W(\epsilon_1, \epsilon_2, \epsilon_3, D^-)}{\partial \lambda_2} \\ s_3 = \frac{\partial W(\epsilon_1, \epsilon_2, \epsilon_3, D^-)}{\partial \lambda_3} \end{cases} \quad (4)$$

$$E^- = \frac{\partial W(\epsilon_1, \epsilon_2, \epsilon_3, D^-)}{\partial D^-} \quad (5)$$

2 Ogden 弹性应变能

2.1 介电弹性体的力学行为

Ogden 弹性应变能是学者 Ogden 在 1972 年最先提出的,常见的弹性应变能模型都是此模型的特例.为使此研究更具有一般性,因此引入多材料常数的 Ogden 弹性应变能函数:

$$W(\lambda_1, \lambda_2, \lambda_3) = \sum_{p=1}^N \frac{\mu_p}{p} (\lambda_1^p + \lambda_2^p + \lambda_3^p - 3) \quad (6)$$

其中 μ_p 是通过实验确定的材料常数,对于不同的介电弹性体材料和不同结构的介电弹性体驱动器, μ_p 是不同的, p 是常数(可以是正数、负数、整数或分数).

把式(6), (1), (3)和(2)代入式(4)和(5)得出介电弹性体三个方向的名义应力和名义电场分别为:

$$s_1 = \sum_{p=1}^N \mu_p \lambda_1^{p-1} \frac{\epsilon_1^{-2} \epsilon_2^{-1} \epsilon_3 D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)]^2} \quad (7)$$

$$s_2 = \sum_{p=1}^N \mu_p \lambda_2^{p-1} \frac{\epsilon_1^{-2} \epsilon_2^{-1} \epsilon_3 D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)]^2} \quad (8)$$

$$s_3 = \sum_{p=1}^N \mu_p \lambda_3^{p-1} \frac{\epsilon_1^{-1} \epsilon_2^{-1} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)]^2} \quad (9)$$

$$E^- = \frac{\epsilon_1^{-1} \epsilon_2^{-1} \epsilon_3 D^-}{[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)]} \quad (10)$$

对介电弹性体三个方向上分别施加载荷 P_1, P_2 和 P_3 ,在变形前的情况下,介电弹性体三个方向上的尺寸分别为 l_1, l_2 和 l_3 ,变形后的尺寸分别为 L_1, L_2 和 L_3 ,三个方向的拉伸率分别为 λ_1, λ_2 和 λ_3 .定

义介电弹性体三个方向的拉伸率分别为其变形后的尺寸除以变形前的尺寸：

$$\lambda_1 = L_1/l_1, \quad \lambda_2 = L_2/l_2, \quad \lambda_3 = L_3/l_3$$

介电弹性体三个方向的名义应力分别为：

$$s_1 = P_1/(l_2 l_3), \quad s_2 = P_2/(l_1 l_3), \quad s_3 = P_3/(l_1 l_2)$$

介电弹性体三个方向的真实应力分别为：

$$\sigma_1 = P_1/(L_2 L_3), \quad \sigma_2 = P_2/(L_1 L_3), \quad \sigma_3 = P_3/(L_1 L_2)$$

由上述定义,介电弹性体三个方向的真实应力可以表达为：

$$\sigma_1 = s_1/(\lambda_2 \lambda_3), \quad \sigma_2 = s_2/(\lambda_1 \lambda_3), \quad \sigma_3 = s_3/(\lambda_1 \lambda_2)$$

代入式(7)-(9)得：

$$\sigma_1 = \frac{\sum_{p=1}^N \mu_p \lambda_1^{p-1} \lambda_2^{-1} \lambda_3^{-1}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} \cdot \frac{\lambda_1^{-2} \lambda_2^{-2} D^{-2}}{b \lambda_1^{-2} \lambda_2^{-2} D^{-2}} \quad (11)$$

$$\sigma_2 = \frac{\sum_{p=1}^N \mu_p \lambda_2^{p-1} \lambda_1^{-1} \lambda_3^{-1}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} \cdot \frac{\lambda_1^{-2} \lambda_2^{-2} D^{-2}}{b \lambda_1^{-2} \lambda_2^{-2} D^{-2}} \quad (12)$$

$$\sigma_3 = \frac{\sum_{p=1}^N \mu_p \lambda_3^{p-1} \lambda_1^{-1} \lambda_2^{-1}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} \cdot \frac{\lambda_1^{-2} \lambda_2^{-2} D^{-2}}{(a+b) \lambda_1^{-2} \lambda_2^{-2} \lambda_3 D^{-2}} \quad (13)$$

定义介电弹性体名义电场是在变形前的情况下,电压除以变形前的尺寸,即 $E^- = U/l_3$,名义电位移是在变形前的情况下,电量除以弹性介电体的变形前面积,即 $D^- = Q/(l_1 l_2)$,与之相对应的弹性介电体真实电场为电压除以变形后的尺寸,即 $E = U/(L_3 L_3) = E^-/\lambda_3$,实际电位移是电量除以变形后的面积,即 $D = Q/(\lambda_1 \lambda_2 l_2 l_2) = D^-/(\lambda_1 \lambda_2)$. 根据上述定义得到下式：

$$E = \frac{D}{[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} \quad (14)$$

上式与真实电位移和真实电场的关系 $D = \epsilon^* E$ 是一致的,这里 ϵ^* 是弹性体的介电常数.

2.2 不可压缩介电弹性体

假设介电弹性体是不可压缩的,即 $\lambda_1 \lambda_2 \lambda_3 = 1$,根据锁等提出的工作,此时名义应力和名义电场的关系简化为下面表达^[35]：

$$s_1 - \frac{s_3}{\lambda_1 \lambda_2} = \frac{\partial W(\lambda_1, \lambda_2, D^-)}{\partial \lambda_1} \quad (15)$$

$$s_2 - \frac{s_3}{\lambda_2 \lambda_1} = \frac{\partial W(\lambda_1, \lambda_2, D^-)}{\partial \lambda_2} \quad (16)$$

$$E^- = \frac{\partial W(\lambda_1, \lambda_2, D^-)}{\partial D^-} \quad (17)$$

根据式(6),此时 Ogden 弹性应变能模型简化为：

$$W(\lambda_1, \lambda_2, \lambda_1^{-1} \lambda_2^{-1}) = \sum_{p=1}^N \frac{\mu_p}{p} (\lambda_1^p + \lambda_2^p + \lambda_1^{-p} \lambda_2^{-p} - 3) \quad (18)$$

引入锁等根据 Wissler 和 Mazza 实验^[36]拟合得到介电弹性体丙烯酸的介电常数与拉伸率的关系^[33]：

$$(\lambda_1, \lambda_2) = [1 + r(\lambda_1 + \lambda_2 - 2)] J^{-1} \quad (19)$$

这里 r 是电致伸缩系数,根据实验数据^[35], $r = -0.053$, $J^{-1} = 4.68 \times 10^{-12}$, ϵ_0 是真空中介电常数, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m.

考虑介电弹性体不可压缩时,整个系统的自由能函数简化为：

$$W(\lambda_1, \lambda_2, D^-) = \sum_{p=1}^N \frac{\mu_p}{p} (\lambda_1^p + \lambda_2^p + \lambda_1^{-p} \lambda_2^{-p} - 3) + \frac{\lambda_1^{-2} \lambda_2^{-2} D^{-2}}{2[1 + r(\lambda_1 + \lambda_2 - 2)] J^{-1}} \quad (20)$$

入式(15),(16)和(17)得介电弹性体机电耦合系统的名义应力和名义电场分别如下：

$$s_1 - \frac{s_3}{\lambda_1 \lambda_2} = \sum_{p=1}^N \mu_p (\lambda_1^{p-1} - \lambda_1^{-p-1} \lambda_2^{-p}) \cdot \frac{\lambda_1^{-3} \lambda_2^{-2} D^{-2}}{[1 + r(\lambda_1 + \lambda_2 - 2)] J^{-1}} - \frac{r \lambda_1^{-2} \lambda_2^{-2} D^{-2}}{2[1 + r(\lambda_1 + \lambda_2 - 2)] J^{-1}} \quad (21)$$

$$s_2 - \frac{s_3}{\lambda_2 \lambda_1} = \sum_{p=1}^N \mu_p (\lambda_2^{p-1} - \lambda_1^{-p-1} \lambda_2^{-p}) \cdot \frac{\lambda_1^{-2} \lambda_2^{-3} D^{-2}}{[1 + r(\lambda_1 + \lambda_2 - 2)] J^{-1}} - \frac{r \lambda_1^{-2} \lambda_2^{-2} D^{-2}}{2[1 + r(\lambda_1 + \lambda_2 - 2)] J^{-1}} \quad (22)$$

$$E^- = \frac{\lambda_1^{-2} \lambda_2^{-2} D^-}{[1 + r(\lambda_1 + \lambda_2 - 2)] J^{-1}} \quad (23)$$

机械力场和电场耦合作用在介电弹性体上,其厚度减小,导致施加同样电压时产生更高电场,这一正反馈过程一直持续下去,当产生的电场超过其临界电场时,机电耦合系统失稳.机械力场对介电弹性体的机电耦合系统的影响由弹性应变能量函数表征,电场对机电耦合系统的影响由电场能量密度函数表征.下面应用具有线性介电常数的电场能量密度函数和简化的 Ogden 弹性应变能函数耦合系

统的自由能函数研究介电弹性体两种特殊载荷情况下的机电稳定性.

3 Ogden 弹性应变能的简化

首先考虑式(6)和(18)中 $N = 1$, 并令 $\mu_1 = 2, \mu_2 = \mu$, 此时把简化后的一个材料常数 Ogden 弹性应变能模型代入式(7)-(9)和(11)-(13)得名义应力和真实应力分别为:

$$s_1 = \mu^{-1} \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{b \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (24)$$

$$s_2 = \mu^{-2} \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{b \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (25)$$

$$s_3 = \mu^{-3} + \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{(a+b) \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (26)$$

$$t_1 = \mu^{-1} \frac{1}{2} \frac{1}{3} \lambda_1^{-1} - \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{b \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (27)$$

$$t_2 = \mu^{-2} \frac{1}{2} \frac{1}{3} \lambda_2^{-1} - \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{b \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (28)$$

$$t_3 = \mu^{-3} \frac{1}{2} \lambda_3^{-1} + \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{(a+b) \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (29)$$

下面考虑把 Ogden 弹性应变能简化为两个材料常数的情况. 令 $N = 2, \mu_2 = -\mu_1 = -2, \mu_1 = C_1, \mu_2 = C_2$, 代入式(7)-(10), 介电弹性体三个方向的名义应力和名义电场分别为:

$$s_1 = C_1 \lambda_1^{-1} - C_2 \lambda_1^{-3} - \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{b \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (30)$$

$$s_2 = C_1 \lambda_2^{-2} - C_2 \lambda_2^{-3} - \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}}$$

$$\frac{b \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (31)$$

$$s_3 = C_1 \lambda_3^{-3} - C_2 \lambda_3^{-3} + \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{(a+b) \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (32)$$

$$E^- = \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} \quad (33)$$

根据上面的推导, 真实应力可以表达为:

$$\sigma_1 = s_1 / (\lambda_2 \lambda_3), \quad \sigma_2 = s_2 / (\lambda_1 \lambda_3), \quad \sigma_3 = s_3 / (\lambda_1 \lambda_2)$$

代入式(30)-(32)得:

$$\sigma_1 = (C_1 \lambda_1^{-1} - C_2 \lambda_1^{-3}) \frac{1}{2} \frac{1}{3} \lambda_2^{-1} \lambda_3^{-1} - \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{b \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (34)$$

$$\sigma_2 = (C_1 \lambda_2^{-2} - C_2 \lambda_2^{-3}) \frac{1}{2} \lambda_3^{-1} - \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{b \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (35)$$

$$\sigma_3 = (C_1 \lambda_3^{-3} - C_2 \lambda_3^{-3}) \frac{1}{2} \lambda_1^{-1} \lambda_2^{-1} + \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^{-1}} - \frac{(a+b) \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)] J^2} \quad (36)$$

当考虑介电弹性体的不可压缩时, 机电耦合系统的名义应力和名义电场分别为:

$$s_1 - \frac{s_3}{\lambda_1 \lambda_2} = C_1 (\lambda_1^{-1} - \lambda_1^{-3} \lambda_2^{-2}) + C_2 (-\lambda_1^{-3} + \lambda_1 \lambda_2) - \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{[1 + r(\lambda_1 + \lambda_2 - 2)] J^{-1}} - \frac{r \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + r(\lambda_1 + \lambda_2 - 2)] J^2} \quad (37)$$

$$s_2 - \frac{s_3}{\lambda_2 \lambda_1} = C_1 (\lambda_2^{-2} - \lambda_2^{-3} \lambda_1^{-2}) + C_2 (-\lambda_2^{-3} + \lambda_2 \lambda_1) - \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{[1 + r(\lambda_1 + \lambda_2 - 2)] J^{-1}} - \frac{r \frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{2[1 + r(\lambda_1 + \lambda_2 - 2)] J^2} \quad (38)$$

$$E^- = \frac{\frac{1}{2} \frac{1}{2} \frac{1}{3} D^{-2}}{[1 + r(\lambda_1 + \lambda_2 - 2)] J^{-1}} \quad (39)$$

事实上, Ogden 弹性应变能可以简化为其它个数材料常数的情况, 根据式(7)-(9)和(11)-(13), 按照上述过程可以得到名义应力和真是应力的表达. 下面研究应用简化为两个材料常数的 Ogden 弹

性应变能时介电弹性体的机电稳定性.

4 机电稳定性分析

4.1 等双轴拉伸

第一种载荷情况,均匀拉伸介电弹性体平面两个方向的薄膜,使两个方向的拉伸率相等,即 $\lambda_1 = \lambda_2 = \lambda$, 由于介电弹性体不可压缩, $\lambda_3 = \lambda^{-2}$, 写出介电弹性体机电耦合系统自由能函数如下:

$$W(\lambda, D^-) = \frac{C_1}{2} (2\lambda^2 + \lambda^{-4} - 3) + \frac{C_2}{2} (2\lambda^{-2} + \lambda^4 - 3) + \frac{\lambda^{-4} D^{-2}}{2[1 + 2r(\lambda - 1)]^{-1}} \quad (40)$$

得出介电弹性体名义电场和名义电位移的关系,令 $s_1(\lambda, D^-) = 0$, 进而可得到 $\partial W(\lambda, D^-) / \partial \lambda = 0$, 代入式(40), 并考虑 $C_2 = kC_1$, k 为与介电弹性体材料和结构有关的常数. 此时, 可推出名义电位移与拉伸率、名义电场与名义电位移、名义电场与拉伸率的关系分别如下:

$$\frac{D^-}{\sqrt{C_1} \lambda^{-1}} = \frac{\sqrt{2[(\lambda^6 - 1) + k(\lambda^8 - \lambda^{-2})]}}{\sqrt{2 + r[1 + 2r(\lambda - 1)]^{-1}} [1 + 2r(\lambda - 1)]} \quad (41)$$

$$\frac{E^-}{\sqrt{C_1} \lambda^{-1}} = \frac{\lambda^{-4}}{1 + 2r(\lambda - 1)} \frac{D^-}{\sqrt{C_1} \lambda^{-1}} \quad (42)$$

$$\frac{E^-}{\sqrt{C_1} \lambda^{-1}} = \frac{2[(\lambda^2 - \lambda^{-8}) + k(1 - \lambda^{-6})]}{\sqrt{2 + r[1 + 2r(\lambda - 1)]^{-1}} [1 + 2r(\lambda - 1)]^{-1}} \quad (43)$$

图 1 给出特殊载荷情况下 ($\lambda_1 = \lambda_2 = \lambda$) 不同介电弹性体材料或介电弹性体结构经历大变形时机电稳定性参数的变化规律. 图 1(a)-(d) 分别为当 $k = 1, 1/2, 1/4, 1/5$ 时^[34], r 取不同值时 ($r = 0.25, -0.053, 0, 1, 2$)^[35], 介电弹性体名义电场和名义电位移的关系. 可以看到, 随着电致伸缩系数 r 的增加, 介电弹性体的名义电场峰值降低, 其机电稳定性降低. 当 $r = 0$, 此时不考虑变形对介电弹性体介电常数的影响, 简化为分析理想介电弹性体的机电稳定性行为, 令 $k = 1/2$, 此时介电弹性体名义电场峰值(临界名义电场)为 $E_{max}^- = 0.9363 \sqrt{C_1} \lambda^{-1}$, 这一结果和刘等得到的结果是相同的^[34]. 考虑电致伸缩系数为 $r = -0.053$ 的情况, 当 $k = 1$ 时, 此时介电弹

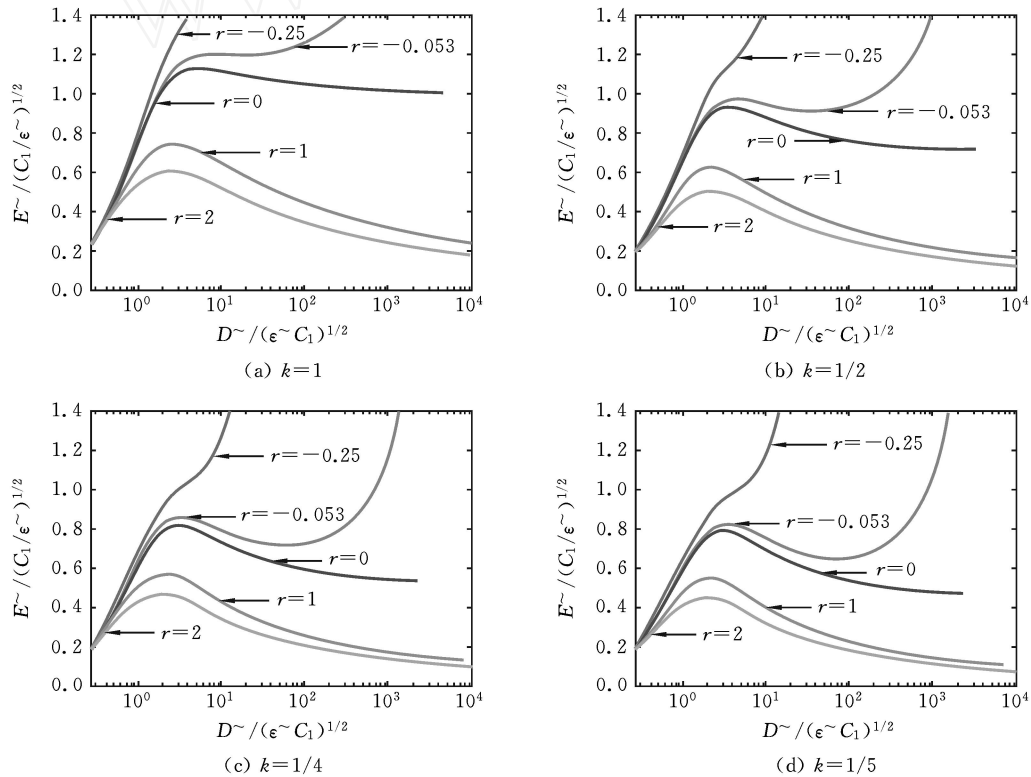


图 1 不同 r 和 k 值时,在拉伸率 $\lambda_1 = \lambda_2 = \lambda$ 情况下,介电弹性体名义电场和名义电位移的关系
Fig. 1 Relationship between the nominal electric displacement and the nominal electric field of dielectric elastomers for various values of r and k , under the stretches are equal biaxial $\lambda_1 = \lambda_2 = \lambda$

性体临界名义电场为 $E_{\max}^- = 1.2104 \sqrt{C_1/\epsilon^-}$; 当 $k = 1/2$ 时, 介电弹性体临界名义电场为 $E_{\max}^- = 0.9762 \sqrt{C_1/\epsilon^-}$; 当 $k = 1/4$ 时, 介电弹性体临界名义电场为 $E_{\max}^- = 0.8480 \sqrt{C_1/\epsilon^-}$; 当 $k = 1/5$ 时, 介电弹性体临界名义电场为 $E_{\max}^- = 0.8212 \sqrt{C_1/\epsilon^-}$. 当电致伸缩系数 r 取其它数值时, 得到的临界名义电场变化规律和上面得到的规律一致. 这说明, 随着 k 值的增大, 介电弹性体临界名义电场增加, 机电稳定性增强. 这与笔者前面工作得到的结论是一致的^[34,39]. 这些结论对指导介电弹性体驱动器、传感器、能量收集器的设计和制造具有重大意义.

图2为在载荷 $\lambda_1 = \lambda_2 = \lambda$ 情况下, 不同介电弹性体经历大变形时名义电场和拉伸率的关系. 在这个分析中, 令材料常数比 $k = 1, 1/2, 1/4, 1/5$ ^[34]. 当 $k = 1/2, r = 0$ 时, 介电弹性体名义电场取得峰值对应的拉伸率(临界拉伸率) $\lambda^c = 1.37$; 这符合笔者前面工作得到的结论^[34]. 若 $r = -0.053$, 介电弹性体名义电场峰值对应较大的临界拉伸率 $\lambda^c = 1.44$, 若

$r = 1$, 介电弹性体名义电场峰值对应较小的临界拉伸率 $\lambda^c = 1.22$, 若 $r = 2$, 介电弹性体名义电场峰值对应最小的临界拉伸率 $\lambda^c = 1.19$. 这与锁等得到的结论是一致的^[28]. 可以看到, 随着 r 的增加, 临界拉伸率减小, 系统机电稳定性降低.

4.2 等厚度拉伸

第二种特殊载荷是均匀拉伸介电弹性体的厚度方向, 即 $\lambda_3 = \lambda$. 假设厚度方向施加的载荷对平面两个方向的影响是等效的, 由于介电弹性体不可压缩, $\lambda_1 = \lambda_2 = \lambda^{-1/2}$, 此时系统的自由能函数可以被简化为:

$$W(\lambda, D^-) = \frac{C_1}{2} (\lambda^{-2} + 2\lambda^{-1} - 3) + \frac{C_2}{2} (\lambda^{-2} + 2\lambda^{-1} - 3) + \frac{2D^-^2}{2[1 + 2r(\lambda^{-1/2} - 1)]^2} \quad (44)$$

类似 §4.1 的推导过程, 令 $s_1(\lambda, D^-) = 0$, 可以得出 $\partial W(\lambda, D^-) / \partial \lambda = 0$, 代入式(44), 同时在计算过程中考虑 $C_2 = kC_1$. 此时, 介电弹性体名义电场、名义电位移和拉伸率的关系可以被简化为如下的形式:

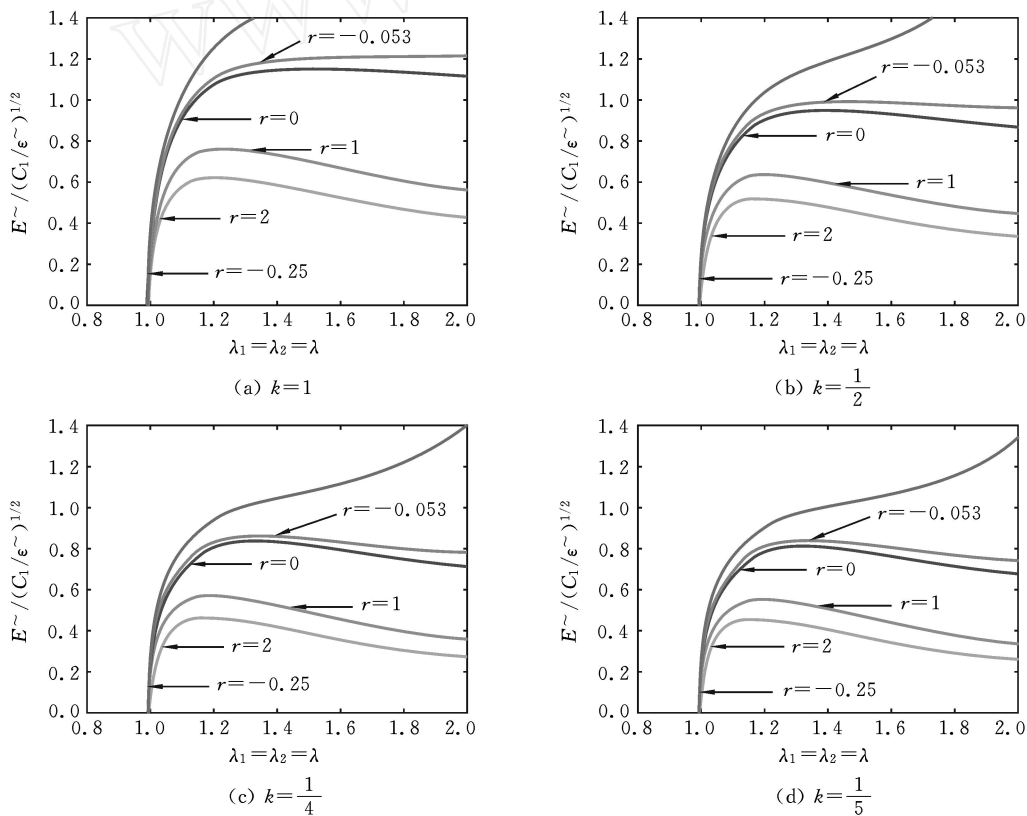


图2 不同 r 和 k 值时, 在拉伸率 $\lambda_1 = \lambda_2 = \lambda$ 情况下, 介电弹性体名义电场和拉伸率的关系

Fig.2 Relationship between the stretch ratio and the nominal electric field of different dielectric elastomers with different values of r and k , under the stretches are equal biaxial $\lambda_1 = \lambda_2 = \lambda$

$$\frac{D^-}{\sqrt{C_1/\epsilon^-}} = \frac{\sqrt{2[(\lambda^-)^{-2} - 1] + k[(\lambda^-)^{-3} - 1]}}{\sqrt{2 + r^{1/2}[1 + 2r((\lambda^-)^{-1/2} - 1)]}} [1 + 2r((\lambda^-)^{-1/2} - 1)]^{-1} \quad (45)$$

$$\frac{E^-}{\sqrt{C_1/\epsilon^-}} = \frac{2}{1 + 2r((\lambda^-)^{-1/2} - 1)} \frac{D^-}{\sqrt{C_1/\epsilon^-}} \quad (46)$$

$$\frac{E^-}{\sqrt{C_1/\epsilon^-}} = \frac{\sqrt{2[(\lambda^-)^{-2} - 5] + k[(\lambda^-)^{-4} - 1]}}{\sqrt{2 + r^{1/2}[1 + 2r((\lambda^-)^{-1/2} - 1)]}} [1 + 2r((\lambda^-)^{-1/2} - 1)]^{-1} \quad (47)$$

图 3 是介电弹性体的厚度方向在承受 $\lambda_3 = 1$ 载荷情况时,其名义电场和名义电位移的关系.可以看到,随着电致伸缩系数 r 的增加,介电弹性体的临界名义电场降低,具有这种性质的介电弹性体材料或者结构的机电稳定性降低.同 §4.1 类似,不考虑变形对介电弹性介电常数的影响 ($r = 0$),当 $k = 1/5$ 时,介电弹性体机电耦合系统的临界名义电场为:

$E_{\max}^- = 0.7923 \sqrt{C_1/\epsilon^-}$,这与刘等得到的结论是一致的^[34].考虑 $r = 1$ 时,当材料常数比 $k = 1, 1/2, 1/4, 1/5$ 时,经历大变形的介电弹性体临界名义电场分别为:

$$E_{\max}^- = 0.7548 \sqrt{C_1/\epsilon^-}, \quad 0.6311 \sqrt{C_1/\epsilon^-}, \\ 0.5604 \sqrt{C_1/\epsilon^-}, \quad 0.5450 \sqrt{C_1/\epsilon^-}$$

图 4 是在 $\lambda_3 = 1$ 况下,介电弹性体名义电场和拉伸率的关系.列举几个特殊情况下的临界拉伸率,令 $k = 1/4$,当 $r = 0$ 时,介电弹性体名义电场取得峰值对应的临界拉伸率 $\lambda_c^- = 0.57$;当 $r = -0.053$ 时,介电弹性体临界拉伸率 $\lambda_c^- = 0.54$,当 $r = 1$ 时,其临界拉伸率 $\lambda_c^- = 0.69$,当 $r = 2$ 时,临界拉伸率 $\lambda_c^- = 0.72$.比较上述数据可得,随着电致伸缩系数 r 的增加,临界拉伸率是增加的.

5 介电弹性体机电稳性能参数

这一节中,研究两种不同拉伸率情况下 ($\lambda_1 = 2, \lambda_2 = 1, \lambda_3 = 1$),介电弹性体机电稳定性性能参数的变化规律(临界名义电场,临界拉伸率,临界面积应变,

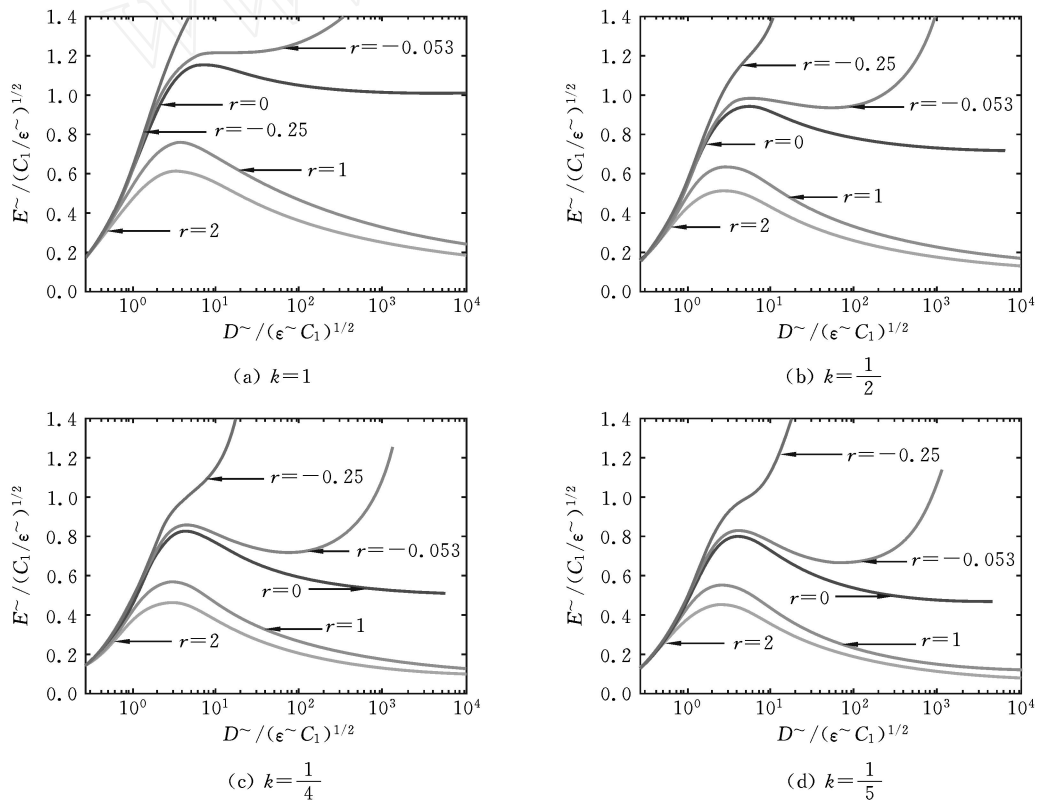


图 3 不同 r 和 k 值时,在拉伸率 $\lambda_3 = 1$ 情况下,介电弹性体机电耦合系统稳定性行为

Fig. 3 Illustrates the stability performance of different dielectric elastomers materials and fabrics in another special loading condition, namely $\lambda_3 = 1$

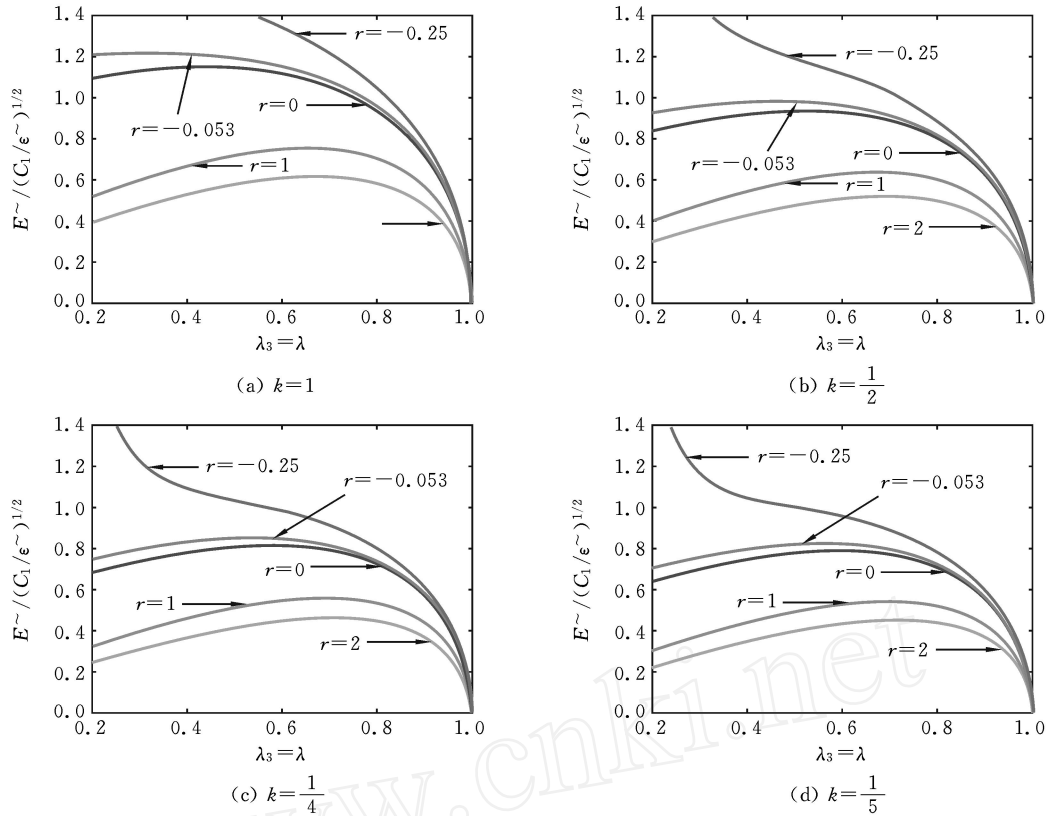


图4 不同 r 和 k 值时,在拉伸率 $\lambda_3 = \lambda$ 情况下,介电弹性体名义电场和拉伸率的关系
 Fig. 4 Relationship between the stretch ratio and the nominal electric field of different dielectric elastomers with different values of r and k , $\lambda_3 = \lambda$

临界厚度应变). 介电弹性体临界名义电场是机电耦合系统从稳定状态到不稳定状态的临界值,是衡量一个介电弹性体机电耦合系统的稳定性高低的重要参数,临界名义电场越高,系统机电稳定性越高. 临界拉伸率是系统达到临界名义电场对应的拉伸率,它是介电弹性体在机械力和电场力耦合作用下能达到的最大拉伸变形. 类似的,临界面积应变,临界厚度应变是介电弹性体在机械力和电场力耦合作用下能达到的最大面积变形、厚度变形.

介电弹性体临界名义电场和名义应力由方程(4)给出,临界拉伸率可以令名义应力等于0得出. 定义临界面积应变和临界厚度应变与拉伸率的关系分别如下:

$$S = \frac{2}{c} \tag{48}$$

$$h = 1 - \frac{1}{2} \tag{49}$$

5.1 等双轴拉伸

图5-8展示以材料常数比 k 为变量,不同电致伸缩系数时,临界名义电场,临界拉伸率,临界面

积应变,临界厚度应变的变化规律. 可以看到,随着材料常数比 k 的增加,临界名义电场、临界拉伸率、临界面积应变和临界厚度应变均增加,这说明, k 值越大的介电弹性体其机电稳定性越高,在整个机电耦合系统稳定区域内达到的临界拉伸率越大,最大

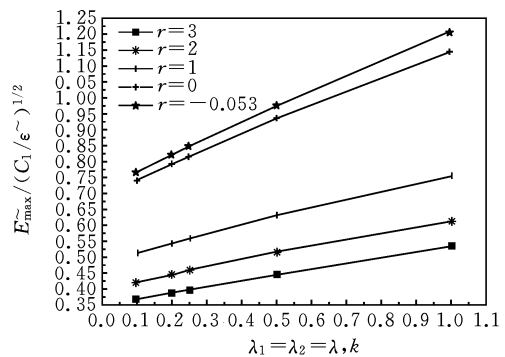


图5 $\lambda_1 = \lambda_2 = \lambda$ 时,电致伸缩系数 r 不同的临界名义电场
 Fig. 5 The critical nominal electric field when electrostriction coefficients r takes different values under the loading condition $\lambda_1 = \lambda_2 = \lambda$

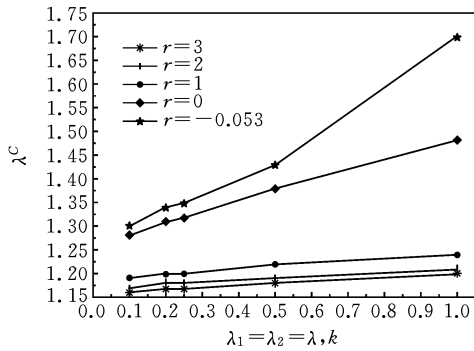


图 6 $\lambda_1 = \lambda_2 =$ 时,电致伸缩系数 r 不同时的临界拉伸率
 Fig. 6 The critical stretch ratio when electrostriction coefficients r takes different values under the loading condition $\lambda_1 = \lambda_2 =$

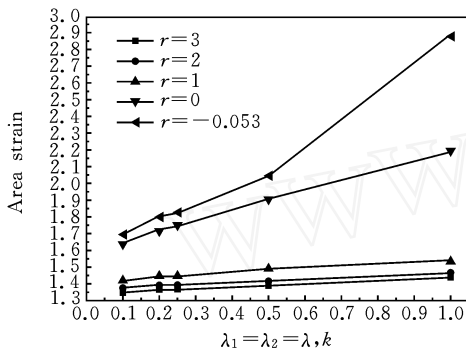


图 7 $\lambda_1 = \lambda_2 =$ 时,电致伸缩系数 r 不同时的临界面积应变
 Fig. 7 The critical area strain when the electrostriction coefficients r takes different values under the loading condition $\lambda_1 = \lambda_2 =$

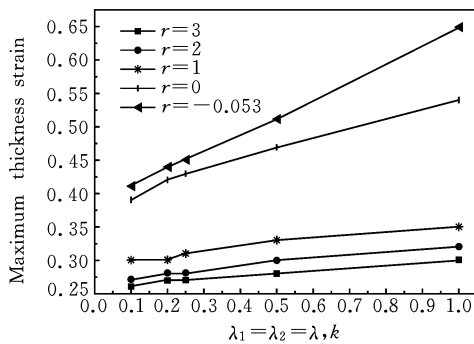


图 8 $\lambda_1 = \lambda_2 =$ 时,电致伸缩系数 r 不同时的临界厚度应变
 Fig. 8 The critical thickness strain when electrostriction coefficients r takes different values under the loading condition $\lambda_1 = \lambda_2 =$

面积应变越大,厚度应变越大,这与笔者前面工作结论一致^[34]。随着电致伸缩系数 r 的增加,临界名义电场、临界拉伸率、临界面积应变和临界厚度应变均减小,这说明, r 越大的介电弹性体机电稳定性越低,在整个系统机电稳定区域内能达到的临界拉伸率越小,最大面积应变越小,厚度应变越小,这与锁等最近工作得到的结论是一致的^[35]。因此,为制造性能更加优异的介电弹性体驱动器、传感器、能量收集器需要选用 k 值大的介电弹性体材料,同时兼顾电致伸缩系数 r 更小。

5.2 等厚度拉伸

图 9-12 为另一种特殊载荷 $\lambda_3 =$ 情况下,介电弹性体机电稳定性参数变化规律。当 k 增加时,经历大变形的介电弹性体临界名义电场,临界面积应变,临界厚度应变均增加,而临界拉伸率降低。这说明, k

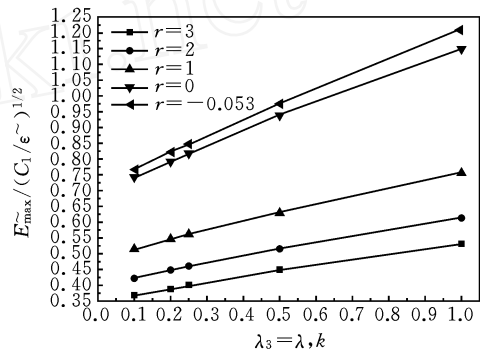


图 9 $\lambda_3 =$ 时,电致伸缩系数 r 不同时的临界名义电场
 Fig. 9 The critical nominal electric field when electrostriction coefficients r takes different values under the loading condition $\lambda_3 =$

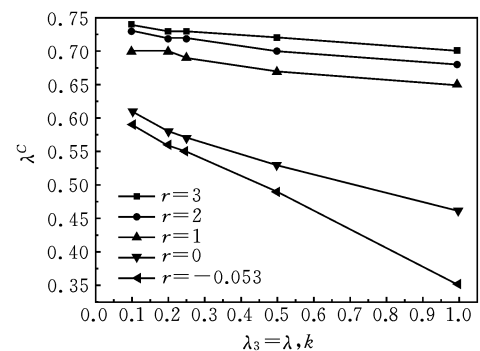


图 10 $\lambda_3 =$ 时,电致伸缩系数 r 不同时的临界拉伸率
 Fig. 10 The critical stretch ratio when electrostriction coefficients r takes different values under the loading condition $\lambda_3 =$

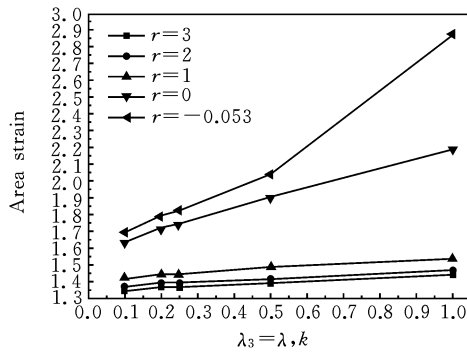


图 11 $\lambda_3 =$ 时,电致伸缩系数 r 不同的临界面积应变

Fig.11 The critical area strain when the electrostriction coefficients r takes different values under the loading condition $\lambda_3 =$

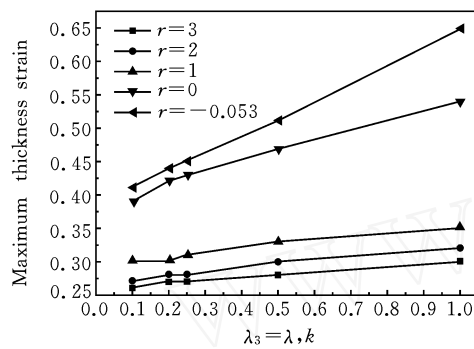


图 12 $\lambda_3 =$ 时,电致伸缩系数 r 不同的临界厚度应变

Fig.12 The critical thickness strain when electrostriction coefficients r takes different values under the loading condition $\lambda_3 =$

值越大的介电弹性体不同材料和结构,其机电稳定性越高,能达到的临界拉伸率越小,最大面积应变越大,厚度应变越大.随着电致伸缩系数 r 增加,临界名义电场,临界面积应变,临界厚度应变均减小,临界拉伸率升高, r 越大的介电弹性体稳定性越低.这些结论与 §5.1 得到的结论是一致的.

综合上述两种特殊载荷情况,为了提高介电弹性体机电耦合系统的稳定性,我们可以选择材料常数比 k 值大和电致伸缩系数 r 值小的介电弹性体材料或介电弹性体结构.

6 结论

本文应用多材料常数的 Ogden 弹性应变能函数和线性介电常数的电场能量密度函数耦合分析介电弹性体的力学行为和机电稳定性,给出两种特殊加载情况下的名义电场和名义电位移,及稳定性参数的变化规律.结果表明,材料常数比 k 值越大及电

致伸缩系数 r 越小的介电弹性体材料或介电弹性体结构更趋稳定.这些有益于我们深入理解介电弹性体的机电稳定性行为和更好指导介电弹性体器件的设计.

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CONSTITUTIVE RELATION ELECTROMECHANICAL STABILITY ANALYSIS OF OGDEN TYPE DIELECTRIC ELASTOMER WITH LINEAR PERMITTIVITY

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Abstract The Ogden elastic strain energy function with multiple material constants is applied to analyze the mechanical performance of dielectric elastomers. The electromechanical stability is investigated in detail. Numerical results show that the proper improvement of material coefficients including the material constant ratio and the electrostriction coefficient can enhance the stability performance of the dielectric elastomer or the dielectric elastomer structure. All these results are helpful in understanding further the stability performance and design of actuators based on dielectric elastomer.

Key words electromechanical stability, dielectric elastomer, Mooney-Rivlin elastic strain energy function, linear permittivity