# A Novel Bistable Hybrid Composite Laminate

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## SUMMARY

A bistable unsymmetric hybrid composite laminate with quite high stiffness and large shape change is presented. Rayleigh-Ritz method is used to predict the cured shape and the predited results are well agreed with the experimentals. The critical loads switching between different shapes are tested. It shows that the critical load for hybrid composite laminates increases greatly (up to 10 times) compared with the pure fiber reinforced polymer matrix composite laminates. The influence of different geometric and material properites on the bistable shape is discussed. It reveals that the present hybrid bistable laminate is more designable and miscellaneous.

Keywords: Unsymmetric laminate, Hybrid composites, Multistable

## Introduction

It is receiving the attention of scientists in developing the multistable structure which enables a number of operational shapes<sup>[1,2]</sup>. It is potentially suitable for a wide variety of the systems, such as morphing aircraft [3-5], deployable structures [6] and mechanical switches. The shape-shifting of aircraft is very challenging because switching between different shapes requires complex, mechanical, power-hungry structures [7, 8]. Recently, bistable structures have been proposed to achieve morphing<sup>[9-12]</sup>. The first important aspect is that large shape change can be accomplished with small energy input and without complicated actuators such as screws, gears or hydraulics. Instead of the power being supplied to elastically deform over its entire range, power is only needed to snap the structure from one stable configuration to another. A reduction in weight of the overall structure is possible, since the whole structure can serve as both the base structure and the control surface. The studies of multistable structures reported in the open literatures are dominated by the bistable unsymmetric composite laminates<sup>[13-17]</sup>. The cured shape has been successfully predicted. The slippage effect has been considered in modelling the cured shape[18]. However, there may be a need to overcome several shortcomings in relying upon laminate asymmetry to produce the required curvatures<sup>[19]</sup>. In general, the thin unsymmetric laminates can offer a larger change in shape with low stiffness. The thick unsymmetric laminates can offer high stiffness with small shape change<sup>[20]</sup>.

Here, a bistable unsymmetric hybrid composite laminate with quite high stiffness and large shape change is presented. The influence of different material properites on the bistable shape is discussed.

## **Cured shape prediction**

The Rayleigh-Ritz method has been successfully employed to predict bistable unsymmetric laminates' cured deformation <sup>[21-25]</sup>. To take into account the large deformations of unsymmetric

laminates, the linear strain-displacement relations must be extended by non-linear terms (see [17, 24]). The principle of minimizing the total potential energy W is used here, given by

$$W = \int_{Vol} \omega dVol$$
, (1)

$$\omega = \frac{1}{2} C_{ijkl} e_{ij} e_{kl} - \beta_{ij} e_{ij} \Delta T$$
<sup>(2)</sup>

Where:  $\omega$  represents strain energy density,  $C_{ijkl}$  denotes the material elastic constants.  $\beta_{ij}$  is the coefficients related to the elastic constants and coefficients of thermal expansion of the material.  $e_{ij}$  are the strains of the materials,  $\Delta T$  is the change of the temperature.

A second order approximation for the out-of-plane displacements of cross-ply laminates is employed though higher order formulae can be got <sup>[17]</sup>. The out-of-plane displacement  $\omega(x, y)$  is assumed to be:

$$\omega(x, y) = \frac{1}{2}(ax^2 + by^2)$$
(3)

Where: a and b are constants  $_{\circ}$  For the description of the in-plane deformations, several approximations can be found in the literature. Here a simple expression from Hyer<sup>[24]</sup> is used, given by:

$$u^{0}(x, y) = cx - \frac{a^{2}x^{3}}{6} - \frac{abxy^{2}}{4}$$
(4)

$$v^{0}(x, y) = dy - \frac{b^{2}y^{3}}{6} - \frac{abx^{2}y}{4}$$
(5)

The constants a, b, c and d are considered as generalized coordinates. The principle of the minimum total potential energy requires the first variation to be zero, which means:

$$\delta W = \left(\frac{\partial W}{\partial a}\right) \delta a + \left(\frac{\partial W}{\partial b}\right) \delta b + \left(\frac{\partial W}{\partial c}\right) \delta c + \left(\frac{\partial W}{\delta d}\right) \delta d \equiv 0$$
(6)

To satisfy this condition, every summand in equation (6) must be zero, which results in a coupled non-linear algebraic equation system in a, b, c and d. The equation group is solved with specially-written mathematical software. To minimize the energy, the second variation of W must be

positive and definite.

## **Bistable Hybrid Composite Laminates**

A number of experiments of unsymmetric hybrid composite laminates were conducted , as seen in Fig.1.



Fig.1 Cured shapes of unsymmetric hybrid composite laminates.

The stacking sequence of first hybrid composite laminate sample with the plane size of  $300 \text{mm} \times 90 \text{mm}$  is 0/0/M/90/90. Here, the number of 0 and 90 indicates the ply angle for the fiber reinforced polymer matrix composite materials. The letter of M denotes the isotropic materials ply. Here a steel plate whith thickness of 0.23 mm, elastic modulus of  $20 \times 10^{10}$ Pa and linear expansion coefficient of  $12 \times 10^{-6}/^{\circ}$ C is used. The polymer matrix composite materials in the experiments are T300/Epoxy with longitudinal elastic modulus E1 of 137.47 GPa, transverse elastic modulus E2 of 10.07 GPa, Possion Ratio v12 of 0.23, the longitudinal linear expansion  $\alpha \perp 0$  f  $0.37 \times 10^{-6}/^{\circ}$ C and transverse linear expansion  $\alpha \ge 0$  f  $24.91 \times 10^{-6}/^{\circ}$ C.

There was a very good agreement between the experimental and analytical shapes. The curvature of first sample from experiment is  $6.16 \text{ m}^{-1}$  and the calculated is  $5.924 \text{ m}^{-1}$ . The second sample with the plane size of 80mm×80mm has the same stacking sequence as the first sample. The experimental curvature is  $6.014 \text{ m}^{-1}$  and the calculated is  $5.798 \text{ m}^{-1}$ . The third sample has a stacking sequence of 0/M/90.The thickness of the middle steel ply is 0.37mm. The experimental curvature is  $3.701 \text{ m}^{-1}$ , and the calculated is  $3.428 \text{ m}^{-1}$ .

The cured curvature of the pure T300/Epoxy laminate with 0/90 stacking sequence is 12.3 m<sup>-1</sup>, and the cured curvature with 0/0/90/90 stacking sequence is 6.02 m<sup>-1</sup>. It has demonstrated that the cured curvature decreases greatly as the thickness increases. That means that the range of shape change decreases greatly with increasing the thickness. On the other hand, the thinner the bistable laminate is, the less the critical load needed to snap it from one stable shape to another. The test for the critical concentrated load at the centre for three bistable laminates is shown in Fig.2.



Fig.2 Three bistable laminates with different materials ply.

The results is listed in Table 1. The very low carrying capacity for pure bistable laminates makes it very limited to be used in morphing structures. However, the critical load for hybrid composite laminates increases greatly (up to 10 times), since the middle ply can considerably enhance the stiffness. At the same time, the hybrid bistable laminate enables a quite large shape change.

Laminates No.	Plane Size	Stacking Sequence	Thickness	Critical Loads	Curvature
	(mm²)		(mm)	(g)	(m⁻¹)
1	300×90	0/0/M/90/90	0.73	590.4	6.16
2	300×100	0/0/90/90	0.50	57.2	6.02
3	300×100	0/90	0.25	23.3	12.34

## Table 1 Critical loads for different bistable laminates

Once the predicted shapes has been successfully compared with the experimental results, the analysis is next used to predict how changes in some of the parameters would influence the shape and behaviour of hybrid composite laminates.

Firstly, the influence of side length of hybrid composite laminates on the bistable configuration is analyzed, as shown in Fig.3 (a) .The theoretical analyses have identified the bifurcation point at which the geometry changes. The shell first deforms into a saddle configuration.Then the equilibrium path bifurcates and the two solution schemes converge to the different geometric shapes. It can be seen that only one group solution exists when the side length is less than some critical value (35 mm), as shown the branch curve AB and A'B' in Fig.3 (a). Here the curvature in x-direction is opposite to that in y-direction. Three group solutions exist when the side length is more than some critical value, as shown with the branches of BD and B'D', BC and B'C', BE and B'E'. The first group solutions as shown with the branches of BD and B'D' mean a saddle shape, which is not in reality. The second and third group of solutions mean existence of snap-through phenomenon. The curvature on the curve BC is 6.49 and the corresponding solution on the curve B'C' is approximately 0. It indicates the cured shape of unsymmetric hybrid composite laminates is similar to a half cylinder.

The similar phenomenon can be found for unsymmetric pure composite laminates<sup>[23,24]</sup>. It should be noted that the middle ply is not limited to be the metal. The idea could also be scaled up or down, and different materials could be used. Therefore, it allows us to select the different middle ply materials to achieve the required bistable configuration.

The influence of the thickness of metallic layer on the bistable configuration is next investigated, as shown in Fig.3 (b). The other parameters are held to be the same as above except the thickness of metallic ply. It can be found that the bifurcation path is different from the above. The shell deforms into a bistable cylinder shape configuration first, and then the equilibrium path converges to the saddle shape.

It can be seen that only one group solution exists when the thickness of metallic layer is greater than a critical value (0.6 mm), as shown with the branches of DF and D'F' in Fig.3 (b). An interesting result can be found that the cured curvature can be greater than the curvature of unsymmetric pure polymer matrix composite laminate as shown with the curve of AB. Unfortunately, it is very difficult to validate the result through the experiment since the suitable parameters including the thickness, the modulus and the expansion coefficient can not be simultaneously satisfied. The cured curvature becomes gradually greater with increasing the thickness from 0 to 0.03 mm. Then the cured curvature becomes gradually less with increasing the thickness from 0.03 to 0.60 mm.

The influence of elastic modulus of metallic ply on the bistable configuration is predicted as shown in Fig.3 (c). It can be seen that only one group solution exists when elastic modulus of metallic ply is greater than a critical value ( $790 \times 10^{10}$ Pa). The cured curvature gets gradually less with elastic modulus from 0 to  $790 \times 10^{10}$ Pa.

Finally, the influence of linear expansion factor of metallic ply on the bistable configuration is investigated, as shown in Fig.3 (d). Interestingly, here two bifurcation points exist. The relationship between the cured curvature and the linear expansion factor of metallic layer is quite linear when it is less than the first critical value  $(-0.4 \times 10^{-6})^{\circ}$ C) and greater than the second critical value  $(-0.3 \times 10^{-6})^{\circ}$ C).

It can be seen that only one group solution exists when linear expansion factor of metallic layer is greater than a critical value ( $-4 \times 10^{-6}$ /°C) and less another critical value ( $-0.3 \times 10^{-6}$ /°C), as shown with the branches of DEF and D'E'F' in Fig.3 (d). The cured curvature gets gradually less with linear expansion factor from  $-5 \times 10^{-6}$ /°C to  $-4 \times 10^{-6}$ /°C. The cured curvature gets gradually less when linear expansion factor is greater than  $-0.3 \times 10^{-6}$ /°C.





Fig.3 Influence of different parameters on cured curvature of hybrid laminates. (a) Influence of the side length; (b) Influence of thickness of metallic ply; (c) Influence of modulus of metallic ply ;(d) Influence of linear expansion factor of metallic ply.

## Conclusions

In conclusion, a novel bistable hybrid composite laminate has been yielded. A morphing aircraft structure needs not noly large shape change but also high carrying capacity. The main benefits of hybrid bistable composite laminate over pure bistable composite laminate have been able to accomplish the two goals simultaneously. The investigations of some geometric and materials properties have revealed that the present hybrid bistable laminate is more designable and miscellaneous.

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