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Stability analysis of dielectric elastomer film actuator

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Dielectric elastomer (DE) is the most promising electroactive polymer material for smart actuators. When a piece of DE film is sandwiched between two compliant electrodes with a high electric field, due to the electrostatic force between the two electrodes, the film expands in-plane and contracts out-of-plane so that its thickness becomes thinner. The thinner thickness results in a higher electric field which inversely squeezes the film again. When the electric field exceeds the critical value, the dielectric field breaks down and the actuator becomes invalid. An elastic strain energy function with two material constants is used to analyze the stability of the dielectric elastomer actuator based on the nonlinear electromechanical field theory. The result shows that the actuator improves its stability as the ratio *k* of the material constants increases, which can be applied to design of actuators. Finally, this method is extended to study the stability of dielectric elastomers with elastic strain energy functions containing three and more material constants.

dielectric elastomer, actuator, electroactive polymer, stability, elastic strain energy function

1 Introduction

Electroactive polymer (EAP) is one of the materials with special mechanical and electrical performance, which can generate several kinds of mechanical responses when exposed to an induced electric field[1−12]. Dielectric elastomers, featuring super large deformation (380%), high elastic energy density (3.4 J/g), high efficiency, high responsive speed, good reliability and durability, are the most promising electroactive polymer material for actuators[13−25].

In recent years, the dielectric elastomer's stability analysis is one of the most popular issues, especially after Suo et al. proposed the electromechanical stability theory of dielectric elastomers[26−34]. In their paper, Suo et al.[28, 29] indicated that any free energy function can be used to analyze the electromechanical stability of dielectric elastomer. For example, they used the elastic strain energy function with one material constant to analyze the stability of an ideal elastic elastomer subject to biaxial stress. The results revealed the relation between the nominal electric displacement and the nominal electric field. It was proved theoretically for the first time that the pre-stretching could enhance the dielectric elastomer's stability. Meanwhile, the critical breakdown electric field strength was predicted using this method[26−29]. Norrisa[30] used the Ogden elastic strain energy model to analyze the elastic elastomer's stability. The relation among critical actual electric field, nominal strain and the pre-stretching ratio of dielectric elastomers was obtained accurately. Meanwhile, as a particular case, the Neo-Hookean model which was a simplified model of Ogden model was introduced to give more concise and accurate results. Further research was done on the stability of Neo-Hookean silicone based elastomer by Díaz-Calleja's group. The Hessian matrix under two special loading conditions was deduced. Furthermore, the stable domain and unstable domain of dielectric elastomers were determined. These results can help us understand the stability performance of Neo-Hooken silicone more thoroughly[31]. An elastic strain

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energy function with two material constants was used to analyze the stability performance of dielectric elastomers by our group. The introduction of material constant ratio k offered a great help in analyzing the stability of various dielectric elastomers[32]. The relation between the nominal electric displacement and nominal electric field of different dielectric elastomers can be derived directly using this model.

This paper takes the dielectric elastomer free energy function as the coupling of elastic strain energy function with two material constants and the electric energy density function to study the stability performance of the dielectric elastomer electromechanical coupling system, as well as the accurate relationship among the nominal electric field, the nominal electrical displacement and the stretch ratio. Such an analysis method is extended to analyze the performance of the dielectric elastomer coupling system with more material constants.

2 Electromechanical coupling theory of dielectric elastomers

In its working condition, dielectric elastomer is subject to an electric field applied by the compliant electrodes spread on the DE film's surfaces. It will contract along the thickness direction due to the electro-static force. Meanwhile an in-plane elongation occurs, developing large strains perpendicular to the voltage direction, then the film becomes thinner[1–10]. It is the process of the coupling effect of the mechanical field and the electric field[26–28]. The energy function of the dielectric elastomer electromechanical coupling system can be written as follows[28]

$$G = L_1 L_2 L_3 W(\lambda_1, \lambda_2, \lambda_3, \tilde{D}) - \boldsymbol{F}_1 \lambda_1 L_1 - \boldsymbol{F}_2 \lambda_2 L_2 - UQ, \quad (1)$$

where $W(\lambda_1, \lambda_2, \lambda_3, \tilde{D})$ is the free energy function, L_1, L_2 and L_3 are the original dimensions of DE, U is the electric voltage, Q is the charge on each surface, $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are the pre-stretch forces, as shown in Figure 1. Let the stretch ratios in the three main directions be λ_1, λ_2, λ_3, respectively. Since the dielectric elastomer is assumed to be incompressible, we have $\lambda_3=1/\lambda_1\lambda_2$. The nominal stress and the nominal electric field are defined as follows

$$s_1 = \frac{\partial W}{\partial \lambda_1}, \quad (2)$$

$$s_2 = \frac{\partial W}{\partial \lambda_2}, \quad (3)$$

$$\tilde{E} = \frac{\partial W}{\partial \tilde{D}}, \quad (4)$$

where s_1 and s_2 are the nominal stresses and can be derived from the un-deformed state by dividing the pre-stretch force by the area before deformation, i.e., $s_1=\boldsymbol{F}_1/(L_2L_3)$ and $s_2=\boldsymbol{F}_2/(L_1L_3)$. L_i, i=1, 2, 3 are the original dimensions of DE, F_i, i=1,2 denote the pre-stretch forces. Similarly, the nominal electric field can be defined as $\tilde{E}=U/L_3$ and the nominal electrical displacement as $\tilde{D}=Q/(L_1L_2)$, while the real electric field defined as $E=U/(\lambda_3L_3)$ and the real electrical displacement as $D=Q/(\lambda_1L_1\lambda_2L_2)$.

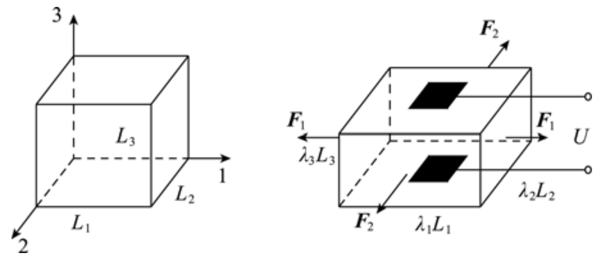

Figure 1 Dielectric elastomer electromechanical coupling system.

Eqs. (2), (3) and (4) are the non-linear algebraic equations which are used to solve the three generalized coordinates λ_1, λ_2 and $\tilde{D}$. Keep $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ as regular vectors, and only let voltage U change. When the voltage is very small, the Hessian matrix is positive definite; when the voltage reaches one critical value $U_{\max}$, the Hessian matrix is not positive and its determinant $\det(\boldsymbol{H}) = 0$. By combining eqs. (2), (3) and (4), the critical $\tilde{E}_{\max}$ can be determined under any pre-stress state.

The Hessian matrix of the dielectric elastomer electromechanical coupling system can be written as follows

$$\boldsymbol{H} = \begin{bmatrix} \dfrac{\partial^2 W}{\partial \lambda_1^2} & \dfrac{\partial^2 W}{\partial \lambda_1 \partial \lambda_2} & \dfrac{\partial^2 W}{\partial \lambda_1 \partial \tilde{D}} \\ \dfrac{\partial^2 W}{\partial \lambda_1 \partial \lambda_2} & \dfrac{\partial^2 W}{\partial \lambda_2^2} & \dfrac{\partial^2 W}{\partial \lambda_2 \partial \tilde{D}} \\ \dfrac{\partial^2 W}{\partial \lambda_1 \partial \tilde{D}} & \dfrac{\partial^2 W}{\partial \lambda_2 \partial \tilde{D}} & \dfrac{\partial^2 W}{\partial \tilde{D}^2} \end{bmatrix}. \quad (5)$$

3 The strain energy function with two material constants

Because the dielectric elastomer is incompressible, only two of the three main stretch ratios λ_1, λ_2, λ_3 are independent. So $W(\lambda_1, \lambda_2, \lambda_3, \tilde{D})$[26–34]

$$W(\lambda_1,\lambda_2,\tilde{D}) = W_0(\lambda_1,\lambda_2) + \frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \tag{6}$$

where $W(\lambda_1, \lambda_2, \tilde{D})$ is the free energy function of the dielectric elastomer electromechanical coupling system, $W_0(\lambda_1, \lambda_2)$ is the elastic strain energy function, and the second term on the right is the electric energy density function. Here the elastic strain energy function adopts the Mooney-Rivlin model with two terms[32, 33].

$$W_0(\lambda_1,\lambda_2) = \frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3) + \frac{G}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3), \tag{7}$$

$$W(\lambda_1,\lambda_2,\tilde{D}) = \frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3) + \frac{G}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3) + \frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \tag{8}$$

where μ and G are material constants determined by experiments. Evidently, the material constants are different for different dielectric elastomer materials (such as BJB TC-A/BC, HS3silicone, CF19-2186 silicone, VHB 4910)[16–22] and different structural dielectric elastomer drives (such as rolling, folding, stacking, flatting)[2–6]. Substituting eqs. (7) and (8) into eqs. (2), (3) and (4) leads to the nominal stress and the nominal electric field respectively as follows

$$s_1 = \frac{\partial W}{\partial \lambda_1} = \mu(\lambda_1 - \lambda_1^{-3}\lambda_2^{-2}) + G(-\lambda_1^{-3}+\lambda_1\lambda_2^2) - \frac{\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{9}$$

$$s_2 = \frac{\partial W}{\partial \lambda_2} = \mu(\lambda_2 - \lambda_2^{-3}\lambda_1^{-2}) + G(-\lambda_2^{-3}+\lambda_2\lambda_1^2) - \frac{\tilde{D}^2}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2}, \tag{10}$$

$$\tilde{E} = \frac{\partial W}{\partial \tilde{D}} = \frac{\tilde{D}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{11}$$

Then the Hessian matrix can be expressed as

$$\boldsymbol{H} = \begin{bmatrix} \mu(1+3\lambda_1^{-4}\lambda_2^{-2}) + G(3\lambda_1^{-4}+\lambda_2^2) + \frac{3\tilde{D}^2}{\varepsilon}\lambda_1^{-4}\lambda_2^{-2} & 2\mu\lambda_1^{-3}\lambda_2^{-3} + 2G\lambda_1\lambda_2 + \frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} \\ 2\mu\lambda_1^{-3}\lambda_2^{-3} + 2G\lambda_1\lambda_2 + \frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & \mu(1+3\lambda_2^{-4}\lambda_1^{-2}) + G(3\lambda_2^{-4}+\lambda_1^2) + \frac{3\tilde{D}^2}{\varepsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} \\ -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix}. \tag{12}$$

Clearly, when G=0, eqs. (9), (10) and (11) become

$$s_1 = \frac{\partial W}{\partial \lambda_1} = \mu(\lambda_1 - \lambda_1^{-3}\lambda_2^{-2}) - \frac{\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{13}$$

$$s_2 = \frac{\partial W}{\partial \lambda_2} = \mu(\lambda_2 - \lambda_2^{-3}\lambda_1^{-2}) - \frac{\tilde{D}^2}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2}, \tag{14}$$

$$\tilde{E} = \frac{\partial W}{\partial \tilde{D}} = \frac{\tilde{D}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{15}$$

And the Hessian matrix can be simplified as

$$\boldsymbol{H} = \begin{bmatrix} \mu(1+3\lambda_1^{-4}\lambda_2^{-2}) + \frac{3\tilde{D}^2}{\varepsilon}\lambda_1^{-4}\lambda_2^{-2} & 2\mu\lambda_1^{-3}\lambda_2^{-3} + \frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} \\ 2\mu\lambda_1^{-3}\lambda_2^{-3} + \frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & \mu(1+3\lambda_2^{-4}\lambda_1^{-2}) + \frac{3\tilde{D}^2}{\varepsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} \\ -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix}. \tag{16}$$

Evidently, the condition of G=0 is the case which has been studied by Zhao and Suo[26].

4 Stability analysis

Pre-stretch uniformly the DE film, such that the stretch ratios in the two directions are equal, i.e.,

$$s_1 = s_2 = s, \tag{17}$$

$$\lambda_1 = \lambda_2 = \lambda. \tag{18}$$

Substituting eqs. (17) and (18) into eqs. (9), (10) and (11), we have the dimensionless quantities as follows

$$\frac{D^{\sim}}{\sqrt{\mu\varepsilon G}} = \sqrt{\frac{\lambda^6 - 1}{G} + \frac{\lambda^8 - \lambda^2}{\mu} - \frac{s}{\mu G}\lambda^5}, \tag{19}$$

$$\frac{E^{\sim}}{\sqrt{\mu G / \varepsilon}} = \sqrt{\frac{\lambda^{-2} - \lambda^{-8}}{G} + \frac{1 - \lambda^{-6}}{\mu} - \frac{s}{\mu G}\lambda^{-3}}. \tag{20}$$

Let μ=kG, where k is constant. Then eqs. (19) and (20) become

$$\frac{D^{\sim}}{\sqrt{\varepsilon G}} = \sqrt{k(\lambda^6 - 1) + \lambda^8 - \lambda^2 - \frac{s}{G}\lambda^5}, \tag{21}$$

$$\frac{E^{\sim}}{\sqrt{G / \varepsilon}} = \sqrt{k(\lambda^{-2} - \lambda^{-8}) + 1 - \lambda^{-6} - \frac{s}{G}\lambda^{-3}}. \tag{22}$$

When k is assigned different constants, the stability of DE materials can be analyzed by considering the pre-stretch ratio as variant. Figure 2 shows the relation ship between $D^{\sim}/\sqrt{\varepsilon G}$ and $E^{\sim}/\sqrt{G/\varepsilon}$ when k=1, 2, 2.5 and 5, respectively. In each case, s/G takes different values such as 0, 1, 2, 3, 4, 5, and $E^{\sim}$ reaches its peak values. Before $E^{\sim}$ reaches its peak value, the Hessian matrix is positive definite while after the peak value, the Hessian matrix is negative definite. Simply speaking, in the peak point, det($\boldsymbol{H}$) =0. As the value s/G increases, the nominal electric field decreases under any constant k, which implies that increasing the prestretch ratio can improve the stability of the DE actuator.

Given that k=2,4,5, the maximum value of the nominal electric voltage $E^{\sim}_{\max} = 0.93\sqrt{2G/\varepsilon}$, $0.817\sqrt{4G/\varepsilon}$, $0.792\sqrt{5G/\varepsilon}$. If $G = 0.25\times10^6$ Pa, $\varepsilon = 4\times10^{-11}$ F/m, then $E^{\sim}_{\max} \approx 1.04\times10^8$, 1.29×10^8 and 1.40×10^8 V/m, which are approximate to the magnitudes of the reported breakdown fields[26]. Furthermore, in this case we obtain the critical stretch $\lambda^C = 1.37, 1.32, 1.30$, and the corresponding strain in the thickness direction equals 46%, 42%, 41%, which agrees the fact that it can not exceed 40% of the experimental value[26].

Figure 3 reveals the relation between the two dimensionless quantities λ_1/λ_1^P and $E^{\sim}/\sqrt{G/\varepsilon}$ when k takes different values of 1, 2, 2.5 and 5, respectively, where λ_1^P represents the pre-stretch ratio of dielectric elastomer and λ_1 is the stretch ratio generated by the electric field. As the nominal stress increases, $E^{\sim}/\sqrt{G/\varepsilon}$ will

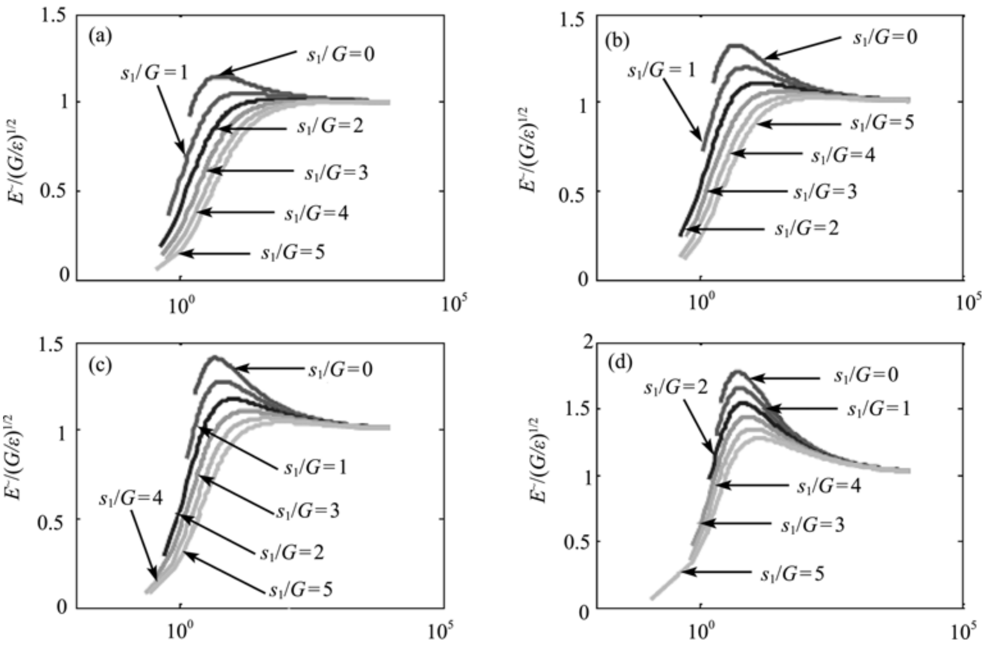

Figure 2 The nominal electric field v.s. the nominal electric displacement when the value k changes. (a) $D^{\sim}/(\varepsilon G)^{1/2}k$=1; (b) $D^{\sim}/(\varepsilon G)^{1/2}k$=2; (c) $D^{\sim}/(\varepsilon G)^{1/2}k$=2.5; (d) $D^{\sim}/(\varepsilon G)^{1/2}k$=5.

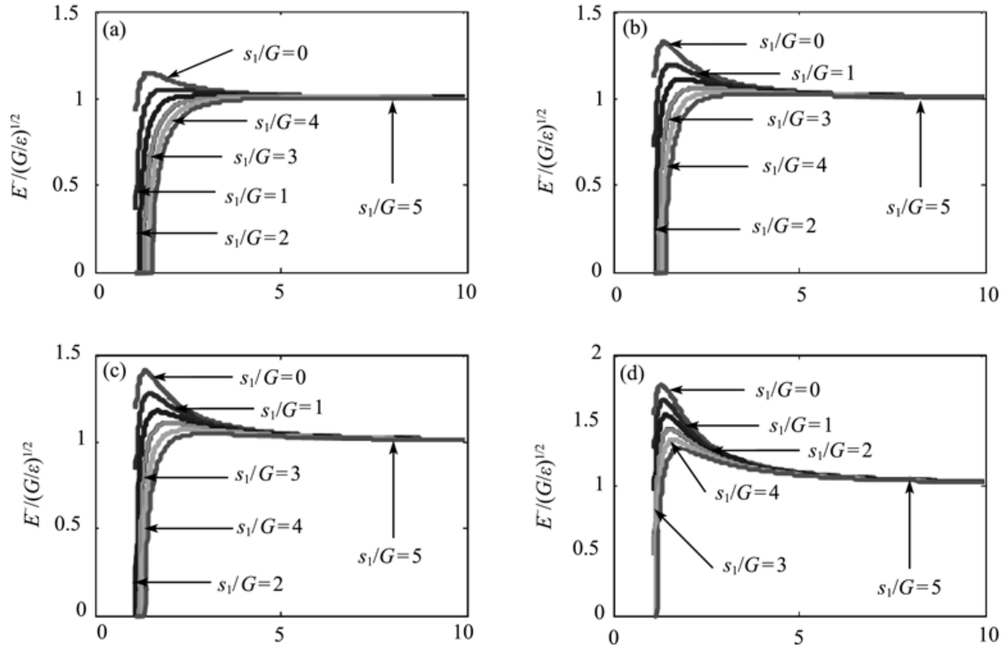

Figure 3 The nominal electric field v.s. the ratio λ_1/λ_1^P when k changes. (a) $\lambda_1/\lambda_1^P k=1$; (b) $\lambda_1/\lambda_1^P k=2$; (c) $\lambda_1/\lambda_1^P k=2.5$; (d) $\lambda_1/\lambda_1^P k=5$.

decrease, which implies that the pre-stretch process before electrical activating may enhance material's stability.

The electrostriction experiments on the pre-stretching dielectric elastomer by Pelrine's group showed that when the two in-plane pre-stretch ratios increase from 0% to 500%, the breakdown electric field increases from 18 to 218 MV/m, mounting up 1100%[17]. This means that the electromechanical stability performance is evidently enhanced. Our numeric molding by applying the Mooney-Rivlin elastic strain energy function has obtained the same result as Pelrine's, as well as Suo's conclusions[26,27].

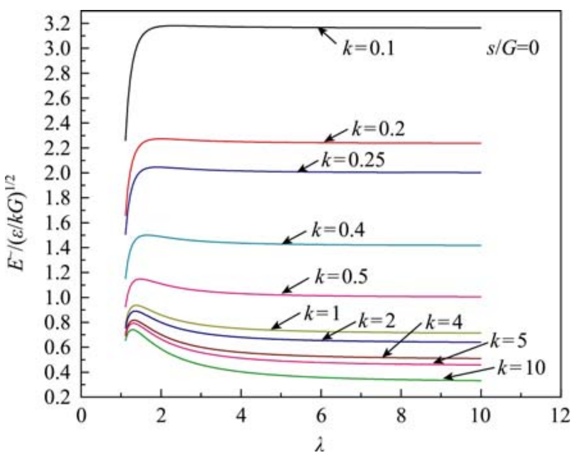

Figure 4 The nominal electric field v.s. the extension ratio when k changes.

Figure 4 reveals the relation between the stretch ratio λ and $E^{\sim}/\sqrt{kG/\varepsilon}$ under the condition of $s/G=0$ when k takes different values. For example, when $k=0.1$, $E^{\sim}_{\max}=3.181\sqrt{0.1G/\varepsilon}$. Clearly, the critical electric field of the dielectric elastomer electromechanical coupling system increases with the increasing of k. This means that with a higher value of k, the system is more stable. These results are useful in the design of dielectric elastomer actuators.

5 Stability analysis based on strain energy function with multi-material-constants

5.1 Elastic strain energy function

Based on the above-related research method, other elastic strain energy functions can be proposed, such as the strain energy functions with three terms: Ishihara- Zahorski function and Yeoh function, etc. The following is to introduce them respectively.

5.1.1 Yeoh function.

$$W_0(\lambda_1,\lambda_2)=\frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)+\frac{G}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^2+\frac{N}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^3. \tag{23}$$

5.1.2 Ishihara-Zahorski function.

$$W_0(\lambda_1,\lambda_2)=\frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)+\frac{G}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)+\frac{N}{2}((\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2})^2-3), \tag{24}$$

μ, G, N in eqs. (23) and (24) are material constants determined by experiments. Note that the values of μ, G, N are different, their same written form is just for the convenience of expression and contrast.

5.1.3 Klosne-Segal function.

$$W_0(\lambda_1,\lambda_2)=\frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)+\frac{G}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)+\frac{N}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)^2+\frac{M}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)^3. \tag{25}$$

5.1.4 Biderman function.

$$W_0(\lambda_1,\lambda_2)=\frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)+\frac{G}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^2+\frac{N}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^3+\frac{M}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3), \tag{26}$$

μ, G, N, M in eqs. (25) and (26) are the material constants also determined by experiments.

5.1.5 Ogden function. The Ogden model includes several material constants proposed by Ogden in 1972, in which the elastic strain energy function is written as

$$W_0(\lambda_1,\lambda_2)=\sum_{p=1}^{N}\frac{\mu_p}{\alpha_p}(\lambda_1^{\alpha_p}+\lambda_2^{\alpha_p}+\lambda_1^{-\alpha_p}\lambda_2^{-\alpha_p}-3), \tag{27}$$

where μ_p is the material constant determined by experiments, and α_p is a constant (positive or negative real number).

5.2 Stability analysis

The constants are different in different elastic strain energy functions for different DE materials. Therefore different elastic strain energy functions can be used to analyse different dielectric elastomer electromechanical coupled systems. The following content is to introduce the analysis methods by applying different elastic strain energy functions. In the followings, we pre-stretch the DE film uniformly so that the stretch ratios in the two orthogonal directions are equal. The following relationships are obtained

$$s_1=s_2=s, \tag{28}$$

$$\lambda_1=\lambda_2=\lambda. \tag{29}$$

5.2.1 Stability analysis based on Yeoh function. The following is the free energy function:

$$W(\lambda_1,\lambda_2,D^{\sim})=\frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)+\frac{G}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^2+\frac{N}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^3+\frac{D^{\sim 2}}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{30}$$

Substituting it into eqs. (2), (3) and (4), we can write the nominal stress and the nominal electric field respectively as

$$s_1=\frac{\partial W}{\partial \lambda_1}=\mu(\lambda_1-\lambda_1^{-3}\lambda_2^{-2})+2G(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)(\lambda_1-\lambda_1^{-3}\lambda_2^{-2})+3N(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^2(\lambda_1-\lambda_1^{-3}\lambda_2^{-2})-\frac{D^{\sim 2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{31}$$

$$s_2=\frac{\partial W}{\partial \lambda_2}=\mu(\lambda_2-\lambda_1^{-2}\lambda_2^{-3})+2G(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)(\lambda_2-\lambda_1^{-2}\lambda_2^{-3})+3N(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^2(\lambda_2-\lambda_1^{-2}\lambda_2^{-3})-\frac{D^{\sim 2}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-3}, \tag{32}$$

$$E^{\sim}=\frac{\partial W}{\partial D^{\sim}}=\frac{D^{\sim}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{33}$$

Dimensionless factors k_1 and k_2 which can be characterized by dielectric elastomer material and structure are induced, $\mu=k_1G=k_2N$, where μ, G and N are the material constants. Then the relationship between nominal electric field and the electrical displacement can be given respectively as

$$\frac{D^{\sim}}{\sqrt{\varepsilon G}}=\left[k_1(\lambda^6-1)+2(2\lambda^2+\lambda^{-4})(\lambda^6-1)+\frac{3k_1}{k_2}(2\lambda^2+\lambda^{-4})^2(\lambda^6-1)-\frac{s}{G}\lambda^5\right]^{1/2}, \tag{34}$$

$$\frac{E^{\sim}}{\sqrt{G/\varepsilon}}=\left[k_1(\lambda^{-2}-\lambda^{-8})+2(2\lambda^2+\lambda^{-4})(\lambda^{-2}-\lambda^{-8})+\frac{3k_1}{k_2}(2\lambda^2+\lambda^{-4})^2(\lambda^{-2}-\lambda^{-8})-\frac{s}{G}\lambda^{-3}\right]^{1/2}. \tag{35}$$

Further, the analytical solution of stability performance for dielectric elastomer electromechanical-coupled system can be calculated based on eqs. (34) and (35), with the similar process mentioned in section 4.

5.2.2 Stability analysis based on VIshihara-Zahorski function. The free energy function can be expressed as

$$W(\lambda_1,\lambda_2,D^{\sim})=\frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)+\frac{G}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)+\frac{N}{2}((\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2})^2-3)+\frac{D^{\sim 2}}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{36}$$

Substituting it into eqs. (2), (3) and (4), we obtain the nominal stress and the nominal electric field respectively:

$$s_1=\frac{\partial W}{\partial\lambda_1}=\mu(\lambda_1-\lambda_1^{-3}\lambda_2^{-2})+G(-\lambda_1^{-3}+\lambda_1\lambda_2^2)+N(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2})(2\lambda_1-2\lambda_1^{-3}\lambda_2^{-2})-\frac{D^{\sim 2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{37}$$

$$s_2=\frac{\partial W}{\partial\lambda_2}=\mu(\lambda_2-\lambda_1^{-2}\lambda_2^{-3})+G(-\lambda_2^{-3}+\lambda_2\lambda_1^2)+N(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2})(2\lambda_2-2\lambda_1^{-2}\lambda_2^{-3})-\frac{D^{\sim 2}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-3}, \tag{38}$$

$$E^{\sim}=\frac{\partial W}{\partial D^{\sim}}=\frac{D^{\sim}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{39}$$

Let $\mu=k_1G=k_2N$, where μ, G and N are the material constants, and k_1 and k_2 are the constants related to the DE material and the composition of the system. Hence, the formulations of the nominal electric displacement and the nominal electric field can be induced respectively.

$$\frac{D^{\sim}}{\sqrt{\varepsilon G}}=\left[k_1(\lambda^6-1)+(\lambda^8-\lambda^2)+\frac{2k_1}{k_2}(2\lambda^2+\lambda^{-4})(\lambda^6-1)-\frac{s}{G}\lambda^5\right]^{1/2}, \tag{40}$$

$$\frac{E^{\sim}}{\sqrt{G/\varepsilon}}=\left[k_1(\lambda^{-2}-\lambda^{-8})+(1-\lambda^{-6})+\frac{2k_1}{k_2}(2\lambda^2+\lambda^{-4})(\lambda^{-2}-\lambda^{-8})-\frac{s}{G}\lambda^{-3}\right]^{1/2}. \tag{41}$$

By eqs. (40) and (41), and taking the tension ratio as the variable with the right material constants k_1 and k_2, the nominal electric field and the electrical displacement can be evaluated and electromechanical stability performance can be analyzed.

5.2.3 Stability analysis based on Klosne-Segal function. The free energy function can be given by

$$W(\lambda_1,\lambda_2,D^{\sim})=\frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)+\frac{G}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)+\frac{N}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)^2+\frac{M}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)^3+\frac{D^{\sim 2}}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{42}$$

Substituting it into eqs. (2), (3) and (4), we obtain the following expressions of the nominal stress and the nominal electric field:

$$s_1=\frac{\partial W}{\partial\lambda_1}=\mu(\lambda_1-\lambda_1^{-3}\lambda_2^{-2})+G(-\lambda_1^{-3}+\lambda_1\lambda_2^2)+2N(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)(\lambda_1\lambda_2^2-\lambda_1^{-3})+3M(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)^2(\lambda_1\lambda_2^2-\lambda_1^{-3})-\frac{D^{\sim 2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{43}$$

$$s_2=\frac{\partial W}{\partial\lambda_2}=\mu(\lambda_2-\lambda_1^{-2}\lambda_2^{-3})+G(-\lambda_2^{-3}+\lambda_1^2\lambda_2)+2N(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)(-\lambda_2^{-3}+\lambda_1^2\lambda_2)+3M(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)^2(-\lambda_2^{-3}+\lambda_1^2\lambda_2)-\frac{D^{\sim 2}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-3}, \tag{44}$$

$$E^{\sim}=\frac{\partial W}{\partial D^{\sim}}=\frac{D^{\sim}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \tag{45}$$

where $\mu=k_1G=k_2N=k_3M$, μ, G, N and M are material constants, and k_1, k_2 and k_3 are constants related to the DE material and the composition of the system. The expressions of the nominal electric field and the electrical displacement can be induced as

$$\frac{D^{\sim}}{\sqrt{\varepsilon G}}=\left[k_1(\lambda^6-1)+(\lambda^8-\lambda^2)+\frac{2k_1}{k_2}(2\lambda^2+\lambda^{-4}-3)(\lambda^8-\lambda^2)+\frac{3k_1}{k_3}(2\lambda^2+\lambda^{-4}-3)^2(\lambda^8-\lambda^2)-\frac{s}{G}\lambda^5\right]^{1/2}, \tag{46}$$

$$\frac{E^{\sim}}{\sqrt{G/\varepsilon}}=\left[k_1(\lambda^{-2}-\lambda^{-8})+(1-\lambda^{-6})+\frac{2k_1}{k_2}(2\lambda^2+\lambda^{-4}-3)(1-\lambda^{-6})+\frac{3k_1}{k_3}(2\lambda^2+\lambda^{-4}-3)^2\times(1-\lambda^{-6})-\frac{s}{G}\lambda^{-3}\right]^{1/2}. \tag{47}$$

Similarly, eqs. (46) and (47) can be used to analyze the stability performance of DE.

5.2.4 Stability analysis based on Bidermanl function. The free energy function can be expressed as below

$$W(\lambda_1,\lambda_2,D^{\sim})=\frac{\mu}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)+\frac{G}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^2+\frac{N}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^3+\frac{M}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3)+\frac{D^{\sim 2}}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{48}$$

Substituting it into eqs. (2), (3) and (4), we can write the nominal stress and the nominal electric field as

$$s_1=\frac{\partial W}{\partial\lambda_1}=\mu(\lambda_1-\lambda_1^{-3}\lambda_2^{-2})+2G(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)(\lambda_1-\lambda_1^{-3}\lambda_2^{-2})+3N(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^2(\lambda_1-\lambda_1^{-3}\lambda_2^{-2})+M(\lambda_1\lambda_2^2-\lambda_1^{-3})-\frac{D^{\sim 2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{49}$$

$$s_2=\frac{\partial W}{\partial\lambda_2}=\mu(\lambda_2-\lambda_1^{-2}\lambda_2^{-3})+2G(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)(\lambda_2-\lambda_1^{-2}\lambda_2^{-3})+3N(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3)^2(\lambda_2-\lambda_1^{-2}\lambda_2^{-3})+M(\lambda_2\lambda_1^2-\lambda_2^{-3})-\frac{D^{\sim 2}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-3}, \tag{50}$$

$$E^{\sim}=\frac{\partial W}{\partial D^{\sim}}=\frac{D^{\sim}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{51}$$

Introduce the dimensionless factors k_1, k_2 and k_3, and let $\mu=k_1G=k_2N=k_3M$, where μ, G, N and M are the material constants. The nominal stress and the nominal electric field of dielectric elastomer can be written as

$$\frac{D^{\sim}}{\sqrt{\varepsilon G}}=\left[k_1(\lambda^6-1)+2(2\lambda^2+\lambda^{-4}-3)(\lambda^6-1)+\frac{3k_1}{k_2}(2\lambda^2+\lambda^{-4}-3)^2(\lambda^6-1)+\frac{k_1}{k_3}(\lambda^8-\lambda^2)-\frac{s}{G}\lambda^5\right]^{1/2}, \tag{52}$$

$$\frac{E^{\sim}}{\sqrt{G/\varepsilon}}=\left[k_1(\lambda^{-2}-\lambda^{-8})+2(2\lambda^2+\lambda^{-4}-3)(\lambda^{-2}-\lambda^{-8})+\frac{3k_1}{k_2}(2\lambda^2+\lambda^{-4}-3)^2(\lambda^{-2}-\lambda^{-8})+\frac{k_1}{k_3}(1-\lambda^{-6})-\frac{s}{G}\lambda^{-3}\right]^{1/2}. \tag{53}$$

Evidently, eqs. (52) and (53) are also capable of analyzing the stability performance.

5.2.5 Stability analysis based on Ogden function. The system free energy function is expressed as follows

$$W(\lambda_1,\lambda_2,D^{\sim})=\sum_{p=1}^{N}\frac{\mu_p}{\alpha_p}(\lambda_1^{\alpha_p}+\lambda_2^{\alpha_p}+\lambda_1^{-\alpha_p}\lambda_2^{-\alpha_p}-3)+\frac{D^{\sim 2}}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{54}$$

The nominal stress and the nominal electric field of the dielectric elastomer electromechanical-coupled system can be expressed respectively by the following equations

$$s_1=\frac{\partial W}{\partial\lambda_1}=\sum_{p=1}^{N}\mu_p(\lambda_1^{\alpha_p-1}-\lambda_1^{-\alpha_p-1}\lambda_2^{-\alpha_p})-\frac{D^{\sim 2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{55}$$

$$s_1=\frac{\partial W}{\partial\lambda_1}=\sum_{p=1}^{N}\mu_p(\lambda_1^{\alpha_p-1}-\lambda_1^{-\alpha_p-1}\lambda_2^{-\alpha_p})-\frac{D^{\sim 2}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{56}$$

$$E^{\sim}=\frac{\partial W}{\partial D^{\sim}}=\frac{D^{\sim}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{57}$$

We postulate that $\mu_1=k_2\mu_2=k_3\mu_3=\cdots=k_N\mu_N$, k_2, k_3, $\cdots$, k_N are material constants. By substituting them into eqs. (55), (56) and (57), the nominal electric field and the nominal electrical displacement can be evaluated. Evidently, they are functions that take the stretch ratio λ as the variable parameter. The relationship between the nominal electric field and the nominal electrical displacement can be derived by changing the value of s/μ_1.

$$\frac{D^{\sim}}{\sqrt{\varepsilon\mu_1}}=\left[(\lambda^{\alpha_1+4}-\lambda^{-2\alpha_1+4})+\frac{1}{k_2}(\lambda^{\alpha_2+4}-\lambda^{-2\alpha_2+4})+\frac{1}{k_3}(\lambda^{\alpha_3+4}-\lambda^{-2\alpha_3+4})+\cdots+\frac{1}{k_N}(\lambda^{\alpha_N+4}-\lambda^{-2\alpha_N+4})-\frac{s\lambda^5}{\mu_1}\right]^{1/2}, \tag{58}$$

$$\frac{E^{\sim}}{\sqrt{\varepsilon\mu_1}}=\left[(\lambda^{\alpha_1-4}-\lambda^{-2\alpha_1-4})+\frac{1}{k_2}(\lambda^{\alpha_2-4}-\lambda^{-2\alpha_2-4})+\frac{1}{k_3}(\lambda^{\alpha_3-4}-\lambda^{-2\alpha_3-4})+\cdots+\frac{1}{k_N}(\lambda^{\alpha_N-4}-\lambda^{-2\alpha_N-4})-\frac{s\lambda^{-3}}{\mu_1}\right]^{1/2}. \tag{59}$$

6 Conclusion

This paper discusses the stability analysis methods of dielectric elastomer by applying the elastic strain energy function with two material constants. The results show that for dielectric material with larger dimensionless constant k, its stability performance is higher. The expressions are educed to analyse the stability performance by applying the elastic strain energy functions with three and more material constants. Evidently, this research offers a great help for design of dielectric elastomer actuators.

1 Baughman R H, Cui C, Zakhidov A A, et al. Carbon nanotubes actuators. Science, 1999, 284(5418): 1340—1344

2 Pelrine R E, Kornbluh R D, Pei Q B, et al. High-speed electrically actuated elastomers with strain greater than l00%. Science, 2000, 287(5454): 836—839

3 Smela E, Inganas O, Lundstrom I. Controlled folding of micrometer-size structures. Science, 1995, 268(5218): 1735—1738

4 Pelrine R E, Kornbluh R D, Joseph J P. Electrostriction of polymer dielectrics with compliant electrodes as a means of actuation. Sensor Actuat A-Phys, 1998, 64(1): 77—85

5 Plante J S, Dubowsky S. On the properties of dielectric elastomer actuators and their design implications. Smart Mater Struct, 2007, 16(2): S227—S236

6 Wissler M, Mazza E. Electromechanical coupling in dielectric elastomer actuators. Sens Actuators A-Phys, 2007, 138(2): 384—393

7 Kofod G, Paajanen M, Bauer S. Self organized minimum energy dielectric elastomer actuators. Appl Phys A-Mater, 2006, 85(2): 141—143

8 Gallone G, Carpi F, Rossi D D, et al. Dielectric constant enhancement in a silicone elastomer filled with lead magnesium niobate-lead titanate. Mater Sci Eng C, 2007, 27(1): 110—116

9 Carpi F, Rossi D D. Improvement of electromechanical actuating performances of a silicone dielectric elastomer by dispersion of titanium dioxide powder. IEEE Trans Dielectr Electr Insul, 2005, 12(4): 835—843

10 Liu Y M, Ren K L, Hofmann H F, et al. Investigation of electrostrictive polymers for energy harvesting. IEEE Trans Ultrason Ferroelectr Freq Control, 2005, 52(12): 2411—2417

11 Zhang Q M, Li H F, Poh M, et al. An all-organic composite actuator material with a high dielectric constant. Nature, 2002, 419(6904): 284—287

12 Liu L W, Fan J M, Zhang Z, et al. Analysis of the novel strain responsive actuators of silicone dielectric elastomer. Adv Mater Res, 2008, 47-50: 298—301

13 Zhang Z, Liu L W, Fan J M, et al. New silicone dielectric elastomers with a high dielectric constant. SPIE, 2008, 6926: 692610(1)—692610(8)

14 Liu Y J, Shi L, Liu L W, et al. Inflated dielectric elastomer actuator for eyeball's movements: Fabrication, analysis and experiments. SPIE, 2008, 6927: 69271A(1)—69271A(8)

15 Liu S F, Park S E, Cross L E, et al. Temperature dependence of electrostriction in rhombohedral $Pb(Zn_{1/3}Nb_{2/3})O_3$-$PbTiO_3$ single crystals. J Appl Phys, 2002, 92(1): 461—467

16 Su J, Xu T B, Zhang S J, et al. An electroactive polymer-ceramic hybrid actuation system for enhanced electromechanical performance. Appl Phys Lett, 2004, 85(6): 1045—1047

17 Kofod G, Sommer-Larsen P, Kronbluh R, et al. Actuation response of polyacrylate dielectric elastomers. J Intell Mater Syst Struct, 2000, 14: 787—793

18 Plante J S, Dubowsky S. Large-scale failure modes of dielectric elastomer actuators. Int J Solids Struct, 2006, 43(25-26): 7727—7751

19 Patrick L, Gabor K, Silvain M. Characterization of dielectric elastomer actuators based on a visco-hyperelastic film model. Smart Mater Struct, 2007, 16(2): 477—486

20 Mockensturm E M, Goulbourne N. Dynamic response of dielectric elastomers. Int J Non-Linear Mech, 2006, 41(3): 388—395

21 Goulbourne N C, Mockensturm E M, Frecker M I. Electro-elastomers: Large deformation analysis of silicone membranes. Int J Solids Struct, 2007, 44(1): 2609—2626

22 Arruda E M, Boyce M C. A three-dimensional constitutive model for the large stretch behavior of rubber elastic materials. J Mech Phys Solids, 1993, 41(2): 389—412

23 Patrick L, Gabor K, Silvain M. Characterization of dielectric elastomer actuators based on a hyperelastic film model. Sensor Actuat A–Phys, 2007, 135(2): 748—757

24 Corona E, Kyriakides S. On the collapse of inelastic tubes under combined bending and pressure. Int J Solids Struct, 1998, 24(5): 505—535

25 Kofoda G, Wirges W, Paajanen M, et al. Energy minimization for self-organized structure formation and actuation. Appl Phys Lett, 2007, 90(8): 081916-1—081916-2

26 Zhao X H, Suo Z G. Method to analyze electromechanical stability of dielectric elastomers. Appl Phys Lett, 2007, 91(6): 061921-1— 061921-3

27 Zhao X H, Hong W, Suo Z G. Electromechanical coexistent states and hysteresis in dielectric elastomers. Phys Rev B, 2007, 76(13): 134113-1—134113-9

28 Suo Z G, Zhao X H, Greene W H. A nonlinear field theory of deformable dielectrics. J Mech Phys Solids, 2008, 56(2): 476—486

29 Zhou J, Hong W, Zhao X H, et al. Propagation of instability in dielectric elastomers. Int J Solids Struct, 2008, 45(13): 3739—3750

30 Norrisa A N. Comment on “method to analyze electromechanical stability of dielectric elastomers”. Appl Phys Lett, 2007, 92(2): 026101-1—026101-2

31 Díaz-Calleja R, Riande E, Sanchis M J. On electromechanical stability of dielectric elastomers. Appl Phys Lett, 2008, 93(10): 101902-1—101902-3

32 Liu Y J, Liu L W, Zhang Z, et al. Comment on “method to analyze electromechanical stability of dielectric elastomers”. Appl Phys Lett, 2008, 93(10): 106101-1—106101-2

33 Liu Y J, Liu L W, Sun S H, et al. Comment on “on electromechanical stability of dielectric elastomers”. Appl Phys Lett, 2009, 94(9): 096101-1—096101-3

34 Leng J S, Liu L W, Liu Y J, et al. Electromechanical stability of dielectric elastomer. Appl Phys Lett, 2009, 94(21): 211901-1— 211901-3