Comment on "Method to analyze electromechanical stability of dielectric elastomers"[Appl. Phys. Lett. 91, 061921 (2007)]

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We would like to thank Zhao and Suo [Appl. Phys. Lett. **91**, 061921 (2007)] for their insights and help on "Method to analyze electromechanical stability of dielectric elastomers;" Suo provided the theory of stability of dielectric elastomers. In this Comment, the nonlinear electromechanical theory for deformable dielectrics described in Suo's paper is applied to analyze the stability of dielectric elastomers. Different from that given in Suo's paper, in the current work, elastic strain energy functional with two material constants is used for the stability analysis. Dielectric materials with different values of k are analyzed, and the results seem to support Suo's theory. These results can be applied in the design of dielectric elastomer actuator and in larger application spectrum.
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The elastic strain energy functional with two material constants can be written as follows:

$$W_0(\lambda_1,\lambda_2) = \frac{C_1}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3) + \frac{C_2}{2}(\lambda_1^{-2}+\lambda_2^{-2} + \lambda_1^2\lambda_2^2-3), \tag{1}$$

where λ_1,λ_2 denote the in-plane principle stretch ratios and C_1 and C_2 are two material constants. According to Suo's theory,[1] the free energy $W(\lambda_1,\lambda_2,\tilde{D})$ is given by

$$W(\lambda_1,\lambda_2,\tilde{D}) = W_0(\lambda_1,\lambda_2) + \frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \tag{2}$$

$$W(\lambda_1,\lambda_2,\tilde{D}) = \frac{C_1}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3) + \frac{C_2}{2}(\lambda_1^{-2}+\lambda_2^{-2} + \lambda_1^2\lambda_2^2-3) + \frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \tag{3}$$

where $\tilde{D}$ is the nominal electric displacement. Taking derivatives, according to Suo's theory,[1–3] we obtain

$$s_1 = \frac{\partial W}{\partial \lambda_1} = C_1(\lambda_1 - \lambda_1^{-3}\lambda_2^{-2}) + C_2(-\lambda_1^{-3}+\lambda_1\lambda_2^2) - \frac{\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{4}$$

$$s_2 = \frac{\partial W}{\partial \lambda_2} = C_1(\lambda_2 - \lambda_2^{-3}\lambda_1^{-2}) + C_2(-\lambda_2^{-3}+\lambda_2\lambda_1^2) - \frac{\tilde{D}^2}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2}, \tag{5}$$

$$\tilde{E} = \frac{\partial W}{\partial \tilde{D}} = \frac{\tilde{D}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \tag{6}$$

where $\tilde{E}$ presents the nominal electric field and ε denotes the electric constant of dielectric elastomer. Note the formulas that appeared in Suo's theory[1] can be recovered if C_2 in Eqs. (4) and (5) is taken to be zero.

The Hessian matrix is invoked herein,

$$H = \begin{bmatrix} C_1(1+3\lambda_1^{-4}\lambda_2^{-2}) + C_2(3\lambda_1^{-4}+\lambda_2^2) + \frac{3\tilde{D}^2}{\varepsilon}\lambda_1^{-4}\lambda_2^{-2} & 2C_1\lambda_1^{-3}\lambda_2^{-3} + 2C_2\lambda_1\lambda_2 + \frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} \\ 2C_1\lambda_1^{-3}\lambda_2^{-3} + +2C_2\lambda_1\lambda_2 + \frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & C_1(1+3\lambda_2^{-4}\lambda_1^{-2}) + C_2(3\lambda_2^{-4}+\lambda_1^2) + \frac{3\tilde{D}^2}{\varepsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} \\ -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix}. \tag{7}$$

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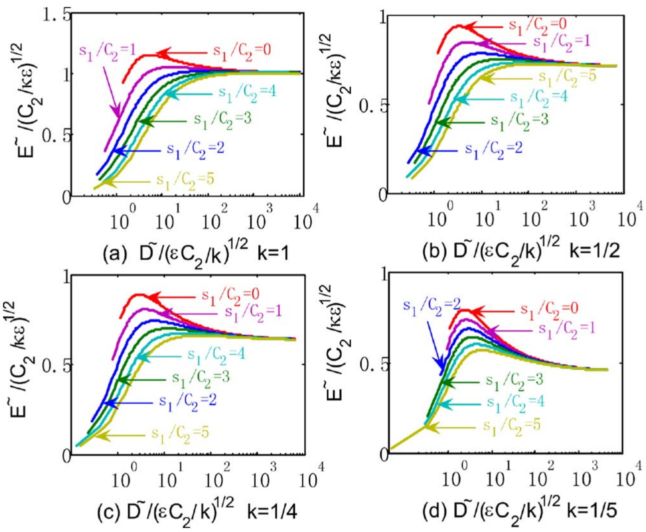

FIG. 1. (Color online) Nominal electric field vs nominal electric displacement at various k's.

The dielectric elastomer is uniformly prestretched such that the in-plane prestretch ratios equal each other, i.e., $s_1 = s_2 = s, \lambda_1 = \lambda_2 = \lambda$. Introducing a dimensionless quantity k, which depends on the material and the activated shape, $C_2 = kC_1$ when C_1 is a constant, as $k=0$, $C_2=0$. The system free energy function is changed to Suo's form.[1] One can nondimensionalize $D^{\sim}$ and $E^{\sim}$ using Eqs. (4)–(6),

$$\frac{D^{\sim}}{\sqrt{\varepsilon C_2/k}} = \sqrt{\lambda^6 - 1 + k(\lambda^8 - \lambda^2) - \frac{ks}{C_2}\lambda^5}, \tag{8}$$

$$\frac{E^{\sim}}{\sqrt{C_2/k\varepsilon}} = \sqrt{\lambda^{-2} - \lambda^{-8} + k(1 - \lambda^{-6}) - \frac{ks}{C_2}\lambda^{-3}}. \tag{9}$$

Figure 1 shows the maximum value of $E^{\sim}/\sqrt{C_2/k\varepsilon}$ at nominal electric displacement $D^{\sim}/\sqrt{\varepsilon C_2/k}$ with different ratios of s/C_2 for $k=1, 1/2, 1/4, 1/5$. For $k=1/2$,[4] it is given as $E_{\max}^{\sim} = 0.936\sqrt{2C_2/\varepsilon}$. Taking $C_2 = 0.25 \times 10^6$ Pa, $\varepsilon = 4 \times 10^{-11}$ F/m, one can further obtain that $E_{\max}^{\sim} \approx 1.04 \times 10^8$ V/m, which is approximately the same magnitude as the reported breakdown field value. Also for $k=\frac{1}{2}$, the critical stretch can be evaluated to be $\lambda^C = 1.37$ and the corresponding strain in the thickness direction as 0.46. To compare, for the same C_1 and ε, the above quantities for the case of $k = \frac{1}{5}$ can be shown to be $E_{\max}^{\sim} = 0.792\sqrt{5C_2/\varepsilon} \approx 1.40 \times 10^8$ V/m, $\lambda^C = 1.30$, and the strain in the thickness direction as 0.41. This indicates that for dielectric elastomer material or dielectric elastomer actuator of different structures, smaller k leads to larger $E_{\max}^{\sim}$, smaller strain in thickness direction, and smaller critical elongation. Based on this observation, the method exposed above is capable of analyzing the stability of dielectric materials having different configurations and k's. Specifically, one may conclude that smaller k enhances markedly the stability.

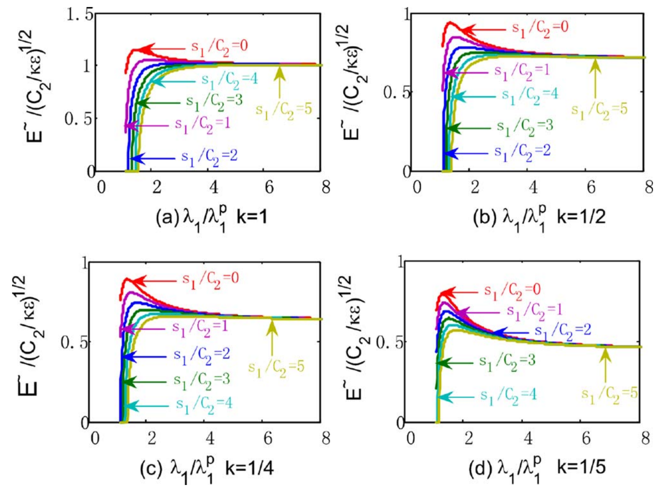

FIG. 2. (Color online) Stretch ratio vs nominal electric field at various k's.

Figure 2 reveals the relation between the two dimensionless quantities λ_1/λ_1^P and $E^{\sim}/\sqrt{C_2/k\varepsilon}$, where λ_1^P is the prestretch ratio of dielectric elastomer and λ_1 is the stretch ratio produced by the electric field. As shown in the figure, when the nominal stress increases, $E^{\sim}/\sqrt{C_2/k\varepsilon}$ decreases. The curve matches well with both the experimental observations and Suo's results.[1]

To summarize, the elastic strain energy functional with two material constants is capable of analyzing the stability of dielectric elastomers. For dielectric material with smaller dimensionless constant k, the corresponding nominal electric breakdown voltage becomes higher, which may be applied in the design of dielectric elastomer actuators.

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