AIP Applied Physics Letters

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Citation: Applied Physics Letters **94**, 096101 (2009); doi: 10.1063/1.3089808
View online: http://dx.doi.org/10.1063/1.3089808
View Table of Contents: http://scitation.aip.org/content/aip/journal/apl/94/9?ver=pdfcov
Published by the AIP Publishing

Comment on "On electromechanical stability of dielectric elastomers" [Appl. Phys. Lett. 93, 101902 (2008)]

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(Received 1 December 2008; accepted 7 February 2009; published online 4 March 2009)

We would like to thank Díaz-Calleja *et al.* [Appl. Phys. Lett. **93**, 101902 (2008)] for their insight and help on "On electromechanical stability of dielectric elastomers;" unstable domain of electromechanical coupling system of neo-Hookean-type silicone was analyzed by Díaz-Calleja *et al.* Different from that given in the paper of Díaz-Calleja, in the current work, the elastic strain energy function with two material constants was used to analyze the stable domain of electromechanical coupling system of Mooney–Rivlin-type silicone, and the results seem to support the theory of Díaz-Calleja. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3089808]

The dielectric elastomer film will encounter electrical breaking-down frequently in its working state due to the coupling effect of electric field and mechanical force field. Zhao and co-workers[1,2] proposed that the free-energy function of any form can be used to analyze the electromechanical stability of dielectric elastomer.

Further research was done on the stability of neo-Hookean silicone based elastomer by Díaz-Calleja *et al.*[3] The stable domain and unstable domain of dielectric elastomers under two special conditions were determined. These results can help us understand the stability performance of neo-Hookean silicone more thoroughly.

Elastic strain energy function with two material constants is used to analyze the stability of Mooney–Rivlin-type silicone under two special loading conditions in purpose of further research the stable domain of this silicone. In this research, the dielectric elastomer is taken to be incompressible.

The free energy with two material constants can be written as the following:[4]

$$W(\lambda_1,\lambda_2,\tilde{D}) = \frac{C_1}{2}(\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3) + \frac{C_2}{2}(\lambda_1^{-2}+\lambda_2^{-2}+\lambda_1^2\lambda_2^2-3) + \frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \tag{1}$$

where λ_1,λ_2 denote the in-plane principle stretching ratios. C_1,C_2 are material constants, which can be determined by experiments, $\tilde{D}$ is the nominal electric displacement, and ε denotes the permittivity of dielectric elastomer. According to the theory of Zhao and co-workers,[1,2] $s_1=\partial W/\partial\lambda_1$, $s_2=\partial W/\partial\lambda_2$, $\tilde{E}=\partial W/\partial\tilde{D}$ are obtained, where $\tilde{E}$ presents the nominal electric field.

Introducing a dimensionless parameter k, which depends on the material and the activated shape, let $C_2=kC_1$. When C_1 is a constant, as $k=0$, $C_2=0$. The system's free-energy function is changed to the form of Zhao and co-workers[1,2] and Díaz-Calleja *at al.*[3] Let $s_1=s_2=0$, $\lambda_1=\lambda_2=\lambda$, Here, nominal electric field and nominal electric displacement are exactly expressed as

$$k\lambda^8+\lambda^6-k\lambda^2-1=\frac{\tilde{D}^2}{\varepsilon C_1}, \tag{2}$$

$$k+\lambda^{-2}+k\lambda^{-6}-\lambda^{-8}=\frac{\tilde{E}^2}{\varepsilon C_1}. \tag{3}$$

According to Eq. (3), the stretching ratio and the maximal nominal electric field $\tilde{E}_{max}$ can be obtained for different dielectric electromers by varying the value of k. For example, in the case of $k=1/2$, $\tilde{E}_{max}=0.936\sqrt{2C_2/\varepsilon}$, and the critical stretch can be evaluated to be $\lambda^C=1.37$; if $k=1/4$, then $\tilde{E}_{max}=0.817\sqrt{4C_2/\varepsilon}$, $\lambda^C=1.32$; if $k=1/5$, then $\tilde{E}_{max}=0.792\sqrt{5C_2/\varepsilon}$, and $\lambda^C=1.30$.

In the first case, we suppose that the in-plane stretch of dielectric elastomer is equal biaxial $\lambda_1=\lambda_2=\lambda$. Let $C_2=kC_1$, $C_1>0$, the free-energy function of electromechanical coupling system can be simplified as follows:

$$W(\lambda,\tilde{D}) = \frac{C_1}{2}(2\lambda^2+\lambda^{-4}-3) + \frac{kC_1}{2}(2\lambda^{-2}+\lambda^4-3) + \frac{\tilde{D}^2}{2\varepsilon}\lambda^{-4}. \tag{4}$$

When C_1 is a constant, as $k=0$, $C_2=0$, The system's free-energy function under the first special loading conditions is changed to the form of Díaz-Calleja *et al.*[4]

The corresponding nominal stress of dielectric elastomer is

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$$s=\frac{\partial W}{\partial \lambda}=2C_1(\lambda-\lambda^{-5})+2kC_1(\lambda^3-\lambda^{-3})-2\frac{D^{\sim 2}}{\varepsilon}\lambda^{-5}. \tag{5}$$

Prestretch is applied to the dielectric elastomer $s>0$; from Eq. (5), the range to keep the dielectric elastomer stable can be obtained,

$$k\lambda^8+\lambda^6-k\lambda^2-1>\frac{D^{\sim 2}}{\varepsilon C_1}. \tag{6}$$

The corresponding Hessian matrix is

$$H=\begin{bmatrix}\dfrac{\partial^2 W}{\partial\lambda^2} & \dfrac{\partial^2 W}{\partial\lambda\,\partial D^{\sim}}\\ \dfrac{\partial^2 W}{\partial\lambda\,\partial D^{\sim}} & \dfrac{\partial^2 W}{\partial D^{\sim 2}}\end{bmatrix}=\begin{bmatrix}2C_1(1+5\lambda^{-6})+6kC_1(\lambda^{-4}+\lambda^2)+10\dfrac{D^{\sim 2}}{\varepsilon}\lambda^{-6} & -\dfrac{4D^{\sim}}{\varepsilon}\lambda^{-5}\\ -\dfrac{4D^{\sim}}{\varepsilon}\lambda^{-5} & \dfrac{1}{\varepsilon}\lambda^{-4}\end{bmatrix}. \tag{7}$$

Due to the steady electromechanical coupling system, Hessian Matrix is positive definite; hence,

$$\begin{vmatrix}2C_1(1+5\lambda^{-6})+6kC_1(\lambda^{-4}+\lambda^2)+10\dfrac{D^{\sim 2}}{\varepsilon}\lambda^{-6} & -\dfrac{4D^{\sim}}{\varepsilon}\lambda^{-5}\\ -\dfrac{4D^{\sim}}{\varepsilon}\lambda^{-5} & \dfrac{1}{\varepsilon}\lambda^{-4}\end{vmatrix}>0. \tag{8}$$

Considering Eqs. (6) and (8), the governing equations of the dielectric elastomer's stability are

$$\begin{cases} y<\sqrt{k\lambda^8+\lambda^6-k\lambda^2-1}\\ y<\sqrt{(3k\lambda^8+\lambda^6+3k\lambda^4+5)/3}\end{cases}, \tag{9}$$

where $y=D^{\sim}/\sqrt{\varepsilon C_1}$.

Figure 1 shows that for different values of k, the steady domain of dielectric elastomer is only below both the two curves (the red and the blue lines, excluding these two curves); simultaneously, all other regions are instable. It can be seen from Fig. 1, when the material constant rate k decreases, that the difference between red and blue curves increases.

In the second case, we take the stretching perpendicular to the plate or layer of polymer as $\lambda_3=\lambda$, $\lambda_1=\lambda_2=\lambda^{-1/2}$ and $C_2=kC_1$ ($C_1>0$); then the free-energy function of electromechanical coupling system can be simplified as follows:

$$W(\lambda_1,\lambda_2,D^{\sim})=\frac{C_1}{2}(\lambda^2+2\lambda^{-1}-3)+\frac{kC_1}{2}(\lambda^{-2}+2\lambda-3)+\frac{D^{\sim 2}}{2\varepsilon}\lambda^2. \tag{10}$$

The system's free-energy function under the other special loading conditions is changed to the form of Díaz-Calleja *et al.*[4]

The nominal stress of the dielectric elastomer is

$$s=\frac{\partial W}{\partial \lambda}=C_1(\lambda-\lambda^{-2})+kC_1(1-\lambda^{-3})+\frac{D^{\sim 2}}{\varepsilon}\lambda. \tag{11}$$

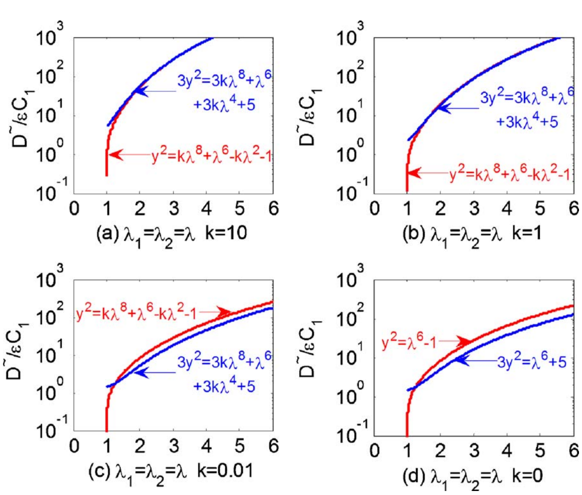

FIG. 1. (Color online) The steady domains of different dielectric elastomers (varying the value of k) subjected to special mechanical load ($\lambda_1=\lambda_2=\lambda$) are illustrated: (a) $k=10$, (b) $k=1$, (c) $k=0.01$, and (d) $k=0$.

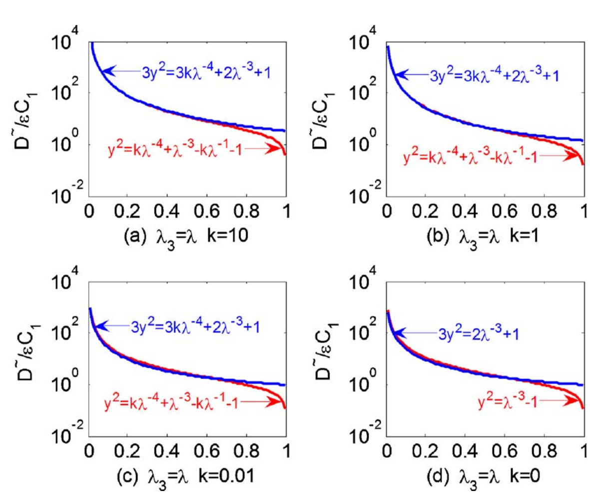

FIG. 2. (Color online) The steady domains of different dielectric elastomers (varying the value of k) subjected to special mechanical load ($\lambda_3=\lambda$) are illustrated: (a) $k=10$, (b) $k=1$, (c) $k=0.01$, and (d) $k=0$.

The corresponding Hessian matrix is

$$H = \begin{bmatrix} \dfrac{\partial^2 W}{\partial \lambda^2} & \dfrac{\partial^2 W}{\partial \lambda\, \partial D^{\sim}} \\ \dfrac{\partial^2 W}{\partial \lambda\, \partial D^{\sim}} & \dfrac{\partial^2 W}{\partial D^{\sim 2}} \end{bmatrix} = \begin{bmatrix} C_1(1+2\lambda^{-3}) + 3kC_1\lambda^{-4} + \dfrac{D^{\sim 2}}{\varepsilon} & \dfrac{2D^{\sim}}{\varepsilon}\lambda \\ \dfrac{2D^{\sim}}{\varepsilon}\lambda & \dfrac{1}{\varepsilon}\lambda^2 \end{bmatrix}. \tag{12}$$

In Eq. (11) $s>0$ and the Hessian matrix [Eq. (12)] is positive definite; the governing equations of the dielectric elastomer's stability in this case are

$$\begin{cases} y > \sqrt{k\lambda^{-4} + \lambda^{-3} - k\lambda^{-1} - 1} \\ y < \sqrt{(3k\lambda^{-4} + 2\lambda^{-3} + 1)/3} \end{cases}, \tag{13}$$

where $y = D^{\sim}/\sqrt{\varepsilon C_1}$.

Figure 2 shows that for different values of k, the steady domain of dielectric elastomer is only between the red and the blue lines (excluding these two curves); simultaneously, all other regions are instable.

In this paper, the free-energy function with two material constants is introduced to analyze the electromechanical stability domain of dielectric elastomers. The stability governing equations and the stability domain of different dielectric elastomers are obtained in the case of different values of k; simultaneously the nominal critical electric field corresponding to the different values of k is calculated. These results provide useful guidelines for the design and fabrication of actuators based on dielectric elastomer.

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